phys522: HW #1

1. Prove Kramer's relation for expectation values of r to power s for the electron wave functions ψ_{nlm} of the hydrogen atom:

   $$\alpha\langle r^s\rangle + \beta a_0\langle r^{s-1}\rangle + \gamma a_0^2\langle r^{s-2}\rangle = 0$$

   where $\alpha = (s+1)/n^2$. $\beta = -(2s+1)$ and $\gamma = (s/4)\left[(2\ell+1)^2 - s^2\right]$

   start with the radial equation. then expectation values $\langle r^s\rangle = \int_0^\infty r^s u(r)^2 dr$ so multiply equation by ur^s and integrate. get a second equation by multiplying the radial equation by $u'r^{s+1}$ and integrate (u' is the derivative with respect to r). Everything else follows by integrating by parts.

2. A particle of mass m is bound in the potential $V(r) = \frac{1}{2}m\omega r^2$ (isotropic three-dimensional harmonic oscillator). Show that the Schrodinger equation can be separated not only in Cartesian coordinates but also in spherical coordinates. Find the degeneracy of each energy level. Show that the ground state has orbital quantum number $\ell = 0$ and that the first excited state has orbital quantum number $\ell = 1$, suggesting that in general the states with the same ℓ are degenerate.

3. (adapted from Commins 8.1)

   The radial equation for the H atom can be transformed into the radial equation of a two-dimensional isotropic harmonic oscillator. To do this, we replace r by $\lambda\rho^2/2$, where λ is a constant to be determined. Also we write $R_{n,\ell} = F(\rho)/\rho$. Show that $F(\rho)$ obeys the radial equation of a two-dimensional harmonic oscillator. Find the constant λ. Use the hydrogen energy $-\mu c^2\alpha^2/2n^2$ and the isolator energy $\hbar\omega(n_0+1)$ where $n_0 = 0, 1, 2\ldots.$ Relate n_0 to n and the two-dimensional angular momentum quantum number m to the orbital quantum number ℓ.