phys522: HW #10

1. Calculate the $\ell = 1$ (P-wave) scattering phase shift δ_1 off of a hard sphere of radius a. Show that in the $ka \to 0$ limit $\delta_1 \approx -(ka)^3/3$ and therefore the P-wave can be neglected compared to the S-wave.

2. In the Born approximation, calculate the scattering amplitude for scattering off of the spherical δ-function potential

$$V(r) = \frac{\gamma\hbar^2}{2m}\delta(r-a)$$

   Find the total Born cross section in the $ka \ll 1$ limit.

   For $\ell = 0$, calculate the scattering off of this same potential by the method of partial waves in the low energy $ka \ll 1$ limit. Compare to the Born result. Determine a condition under which the Born approximation is valid. In this limit, compare to the partial wave result.

3. Consider $l = 0$ (S-wave) scattering off of a finite, attractive potential well, $V(r) =$

$$\begin{aligned} -V_0,&\ r < a \\ 0,&\ r > a \end{aligned}$$

   Solve for the phase shift and total $\ell = 0$ cross section in three cases:

   case 1: $ka \ll 1$ and $\delta_0 \ll 1$, show that σ_o is independent of k

   case 2: $ka \ll 1$ and $k_0 a \approx \pi/2$ where $\hbar k_0 = \sqrt{2m(E - V_0)}$. Show that there is a corresponding bound state with near zero energy.

   case 3: $\delta_0 \approx \pi$, and show that $\sigma_o \approx 0$. Sketch $U(r) = rR$ in this case, and compare to $U(r)$ for $V_0 = 0$. This is called the Ramsauer-Townsend effect and is observed in low energy electron-atom scattering by Nobel gases.