phys522: HW #5

1. Consider a one dimensional (x coordinate) harmonic oscillator with a charge in an electric field which introduces a perturbation,

$$\lambda H' = -q\mathcal{E}x$$

Where the dimensionless constant

$$\lambda = \frac{q\mathcal{E}}{\hbar\omega}\sqrt{\frac{\hbar}{m\omega}}$$

Calculate the first non-vanishing energy correction in perturbation theory. Calculate the leading order correction to the wave function. Solve the problem exactly, and compare to the perturbative result. Show that your result is independent of λ

2. Positronium is a bound state of an electron and positron. Positronium will annihilate to two photons in the ground state, either singlet ${}^1S_0$ or the triplet ${}^3S_1$. The singlet state decays to two photons with a lifetime of 1.25×10^{-10}s. whereas the triplet state is longer lived, having to decay to three photons in 1.4×10^{-7}s. The longer lifetime of the triplet can be understood as follows: the decay rate (inverse lifetime) of the triplet gets a factor for the emission of the additional photon of α. However, it turns out that the calculation gives an additonal purely numerical factor of 0.12, so the triplet rate is down by a factor of only $\approx 10^{-3}$.

Calculate the hyperfine energy splitting for positronium (*give your answer in terms of the electron mass m and the fine structure constant α*). There are two terms, a Fermi contact magnetic moment interaction

$$H_F = -\frac{8\pi}{3}\vec{\mu_+} \cdot \vec{\mu_-}\delta^3\left(\vec{r}\right)$$

and a second term due to one photon exchange

$$H_\gamma = \pi\left(\frac{e}{mc}\right)^2 S^2\delta^3\left(\vec{r}\right)$$

where S is the total spin operator. The photon has charge-parity of -1. The charge-parity of the bound state is equivalent to a combination of spatial-exchange (parity) and spin-exchange. There is, in quantum field theory, the fact that particles have an intrinsic parity and that this is opposite for particle and anti-particle. Therefore, the parity of a particle-aniparticle bound state gets a factor -1^ℓ from the spherical harmonic times an additional factor of -1. Given this, explain the depence on S of this operator as well as the fact that the singlet decays to two photons and the triplet to three photons.

Calculate the splitting frequency in MHz and compare to the current experimental value.