phys522: HW #6

1. Consider a hydrogen atom in and electric field (known as the Stark effect). Ignoring relativistic corrections, calculate the energy correction for $n = 2$ due to the additional interaction,

   $H_1 = -\vec{\mu_{\mathcal{E}}} \cdot \vec{\mathcal{E}}$ where $\vec{\mu_{\mathcal{E}}}$ is the electric dopole moment operator.

   How do the energy corrections depend on electric field strengh?

2. Find the Stark energy eigenstates, and the expectation values of the electric dipole operator for these states. Show that the answer is consistent with the classical expectation.

3. For the next problem we need two expectation values. For hydrogen, calculate $\langle 1/r \rangle$ from Kramer's relation. Calculate $\langle 1/r^2 \rangle$ from the Feynman-Hellmann theorem

   $$\frac{\partial E}{\partial \lambda} = \langle \frac{\partial H}{\partial \lambda} \rangle$$

   where λ is a parameter and the state in the expectation value is an energy eigenstate. For H, use the radial wave function Hamiltonian and treat the quantum number ℓ as a continuous parameter. For the energy, use $n = (n_r + \ell + 1)$ where n_r is the number of radial nodes.

4. Calculate the relativistic kinetic energy correction for hydrogen,

   $$H^K = -\frac{p^4}{8m^3c^2}$$

   Use the trick

   $$\frac{p^2}{2m} = E_0 + \frac{e^2}{r}$$