**phys522: HW #7**

1. Calclulate the spin-orbit correction energy for hydrogen for $n = 2$ in terms of $m$ and $Z\alpha$.

2. Show that the energy due to the Darwin term only effects the $\ell = 0$ states and is given by

$$E_{D,n,0} = \frac{1}{2n^3} mc^2 (Z\alpha)^4$$

   Hint: rewrite as proportional to the divergence of the electric field.

3. In this problem you will show that in general, the relativistic corrections depend only on the quantum numbers $n, j$ where $|\ell - \frac{1}{2}| \leq j \leq \ell + \frac{1}{2}$

   (a) First, show that for the case $\ell > 0$ the total fine structure correction is

$$E^{fs}_{n,j} = a \left[ 3 - \frac{4n}{j + \frac{1}{2}} \right]$$

   where $a \equiv mc^2\alpha^4/8n^4$.

   (b) Next show that for $\ell = 0, j = \frac{1}{2}$ the total fine structure is

$$E^{fs}_{n,\frac{1}{2}} = a\,[3 - 4n]$$

   which is the same as the $\ell > 0, j = \frac{1}{2}$. Therefore, the states $nS_{\frac{1}{2}}$ and $nP_{\frac{1}{2}}$ are degenerate.