phys522: HW #8

1. Work out the Hamiltonian matrix for the complete $n = 2$ Zeeman effect. Take the external magnetic field B^{ext} to be in the z-direction. The full perturbation Hamiltonian for $n = 2$ is $H^Z = H^{SO} + H^B$ where

$$H^{SO} = A\left[3 - \frac{8}{j + \frac{1}{2}}\right]$$

   And with $b \equiv e\hbar B^{ext}/2mc$

$$H^B = b\left(m_\ell + 2m_s\right)$$

   In the notation L_{j,m_j} choose the basis order $|i\rangle, i = 1, 8$

   $S_{\frac{1}{2},\frac{1}{2}}$, $S_{\frac{1}{2},\frac{-1}{2}}$, $P_{\frac{3}{2},\frac{3}{2}}$, $P_{\frac{3}{2},\frac{-3}{2}}$, $P_{\frac{3}{2},\frac{1}{2}}$, $P_{\frac{1}{2},\frac{1}{2}}$, $P_{\frac{3}{2},\frac{-1}{2}}$, $P_{\frac{1}{2},\frac{-1}{2}}$

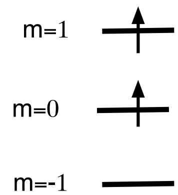
   So for example $P_{\frac{3}{2},\frac{3}{2}} = |3\rangle = |3/2, 3/2\rangle = |1, 1\rangle\,|1/2, 1/2\rangle$

   Write the matrix $\langle i|\left(H^{SO} + H^B\right)|j\rangle$.

   You will need Clebsch-Gordon coefficients which you can look up here in the table. Two pairs of states are mixed by the external magnetic field. Which pairs are they?

2. Determine the spectroscopic term for the ground state of Carbon. First, make a table of all possible values of ℓ, s and j. Include a column for the symmetry of the spatial part and the spin part as $\pm$. Eliminate the rows of the table that are excluded by the Pauli principle. Add a column for the spectrocopic term ${}^{2S+1}L_J$ for all Pauli allowed rows. Finally, apply Hund's rules to determine the ground state.

   You can get the same result using a short-cut (from Gasiorowicz). Make a drawing with horizonal lines ("shelves") corresponding to the values of m_ℓ for the L of the sub-shell. Distribute the electrons as up or down arrows on the shelves, only doubling up on an m_ℓ when necessary and using Hund's rule 1 to maximize the sum of s_z which gives S. Sum the m_ℓ for the shelves to get L as $L = \sum n_i \cdot m_i$ where n_i is the number of electrons on the self. Use Hund's rule 3 to give J. For Carbon the diagram would be,



$S = 1/2+1/2=1$

$L= \sum n\_i*m\_i = 1(1)+1(0)+0(-1) = 1$

Use this technique to find the spectroscopic term for oxygen and manganese.