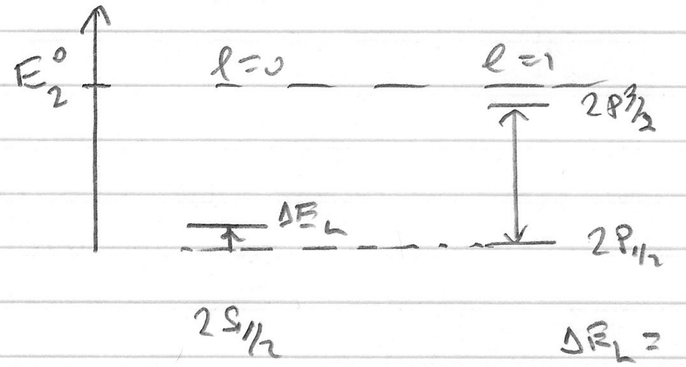
Phys 522

Lamb Shift



$\Delta E_f = 10.9\,\text{GHz}$

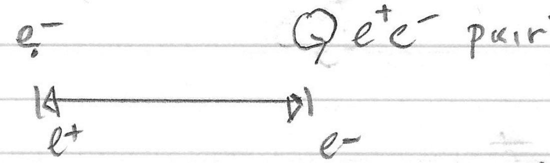
$\Delta E_L = 1.057\,\text{GHz}$

Lamb shift - vacuum fluctuations of Em field

Similar to <u>Darwin term</u> - virtual e^+e^- pairs in vacuum cause electron Zitterbewegung:

$\Delta E \Delta t \sim \hbar$

$\Delta E = 2mc^2$, $\Delta t \sim \dfrac{\hbar}{2mc^2}$



$2\Delta x = 2c\Delta t$, $\Delta x = \dfrac{\hbar}{mc} = \bar{\lambda}$

$\bar{\lambda}$ is Compton wavelength $/2\pi$.

Effective smearing of potential

$$\tilde{V}(\vec{r}) = \int d^3\vec{r}\,'\, f(\vec{r}\,')\, V(\vec{r}+\vec{r}\,')$$

take $f(\vec{r})$ to be Gaussian with $\sigma = \bar{\lambda}$

Expand

$$V(\vec{r}+\vec{r}\,') = V(\vec{r}) + \vec{r}\,'\cdot\vec{\nabla}V\Big|_{\vec{r}} + \frac{1}{2}(\vec{r}\,'\cdot\vec{\nabla})^2 V\Big|_{\vec{r}}$$

smearing integral,

$$\langle \vec{r}\,'\rangle_f = \int d^3\vec{r}\,' f(\vec{r}\,')\, r' = 0$$

So
$$\tilde{V}(r) = V(\vec{r}) + \frac{1}{2}\sum_{i,j=1}^{3}\langle x_i x_j\rangle \frac{\partial^2 V}{\partial x_i x_j}\Big|_{\vec{r}}$$

$\langle x_i x_j\rangle$ cross terms are zero

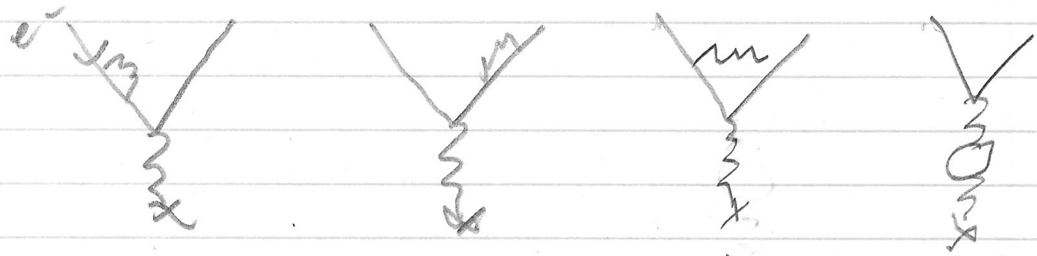
$$\langle x_i^2\rangle = \frac{1}{3}\langle r^2\rangle_f = \frac{1}{3}\bar{\lambda}^2 \quad \text{(Gaussian)}$$

$$\hat{V}(\vec{r}) = V(\vec{r}) + \frac{1}{6}\bar{\lambda}^2\,\nabla^2 V$$

the correction is very nearly Darwin term.

$$\hat{H}_D = \frac{\bar{\lambda}^2}{8}\nabla^2 V$$

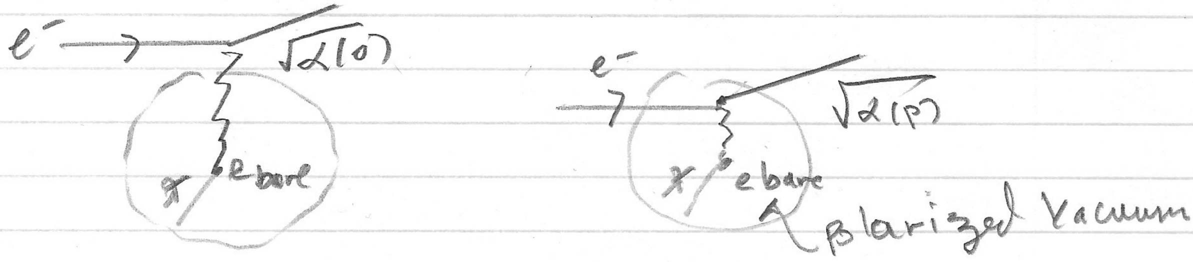
Fluctuations in EM field give amplitudes in perturbation theory that can be represented by Feynman diagrams



mass — mass — anomalous magnetic moment — vacuum polarization

+1017 MHz — +68 MHz — −27 MHz

Picture of QED vacuum: Bare electric charge shielded by vacuum polarization



low energy

high energy

$\alpha(0) = 1/137$

$\alpha(m_Z c^2) = 1/128$

" 91 GeV

Lamb shift PRL 68 1120 (1992)

$\Delta E_L^{exp}/h\,(2S-2P) = 1057.845 \pm 0.009$ MHz

$\Delta E_L^{th}/h\,(2S-2P) = 1057.874 \pm 0.018$ MHz

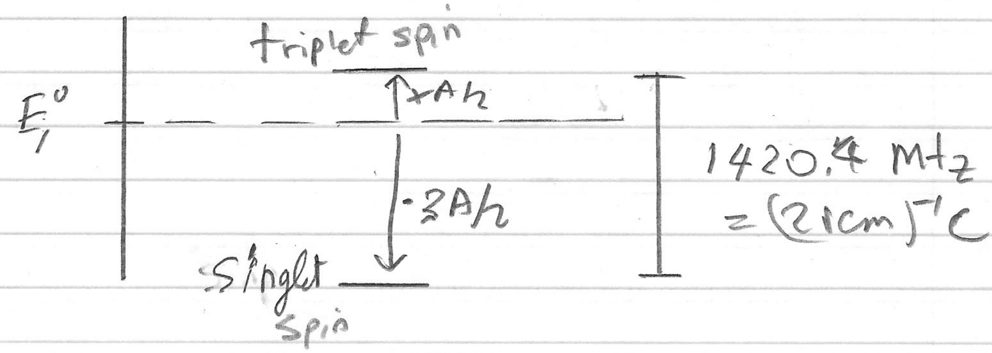
Theory limited by uncertainty of proton radius
Exp limited by irreducible 2p natural line width

Hyperfine Splitting

Electron - proton magnetic moment interaction, called hyperfine interaction.

$$H_{hf} = A \, \vec{\mu}_e \cdot \vec{\mu}_p \, \delta^3(\vec{r})$$

+ small $l \neq 0$ term $\qquad A \equiv$ constant



$$A = \frac{2}{3} mc^2 (Z\alpha)^4 \left(\frac{m_e}{m_p}\right) g_p$$

g_p proton anomalous magnetic moment ~ 5.59

$$\frac{m_e}{m_p} = \frac{1}{1800}$$