Phys 522

Neutrinos

Effective solar fusion process

$$4p \rightarrow He + 2e^{+} + 2\nu_e + 25\,\text{meV}$$

Solar neutrino puzzle.

Calculated experimental detection rate

$$1\ \text{SNU} = 10^{-36}\ \text{events}/(\text{target atom})/\text{sec}$$

Observed in Davis chlorine ^{37}Cl = 3 SNU
Calculated 6 SNU = 2×(observed)

SNO detector (Sudbury Canada)
measured missing rate by observing
all 3 flavors of neutrino. 2015 nobel prize

3 families of quarks and leptons

up	charm	top	quarks
down	strange	bottom	
ν_e	ν_μ	ν_τ	leptons
e	μ	τ	

3 flavors of quarks are produced with associated lepton, interacting with weak force carrier W:

$$W^- \rightarrow \begin{cases} \bar{\nu}_e + e^- \\ \bar{\nu}_\mu + \mu^- \\ \bar{\nu}_\tau + \tau^- \end{cases}$$

detected by production of corresponding charged lepton

$$\nu_e + W^- \rightarrow e^-$$

$$\nu_\mu + W^- \rightarrow \mu^-$$

$$\nu_\tau + W^- \rightarrow \tau^-$$

solar neutrino puzzle solved by flavor oscillation

sun $e^+ \nu_e$ ⇝ earth

infer $\nu_e \rightarrow \nu_\mu$ observe μ^+
infer $\nu_e \rightarrow \nu_\tau$ observe τ^+

ν mixing

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{bmatrix} 3\times 3 \\ \text{mixing} \end{bmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

flavor   mass

3×3 mixing parameterized by
3 angles + phase

Simple two flavor mixing

$$\begin{pmatrix} |\nu_e\rangle \\ |\nu_\mu\rangle \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} |\nu_1\rangle \\ |\nu_2\rangle \end{pmatrix}$$

ν are ultra relativistic

$$E = \sqrt{p^2+m^2} \cong p + \frac{m^2}{2p}$$

$$e^{-iEt/\hbar} = e^{-ipt/\hbar}\, e^{-i\frac{m^2 t}{2p\hbar}}$$

$$|\nu_e\rangle = e^{-ipt/\hbar}\left[e^{-im_1^2 t/2p\hbar} \cos\theta\, |1\rangle + e^{-im_2^2 t/2p\hbar} \sin\theta\, |2\rangle \right]$$

ignore overall phase

$$|\nu_e(t)\rangle = \cos\theta\, |1\rangle + e^{-i\phi_{21}} \sin\theta\, |2\rangle$$

$$\phi_{21} = \frac{m_2^2 - m_1^2}{2p\hbar}\, t \underset{p=E}{=} \left(\frac{\Delta m_{21}^2}{2\hbar E}\right) t$$

then at time t, amplitude to observe flavor state $|\nu_\mu\rangle$

$$|\nu_\mu\rangle = -\sin\theta\,|\nu_1\rangle + \cos\theta\,|\nu_2\rangle$$

$$\langle \nu_\mu | \nu_e\rangle = (-s, c)\begin{pmatrix} c \\ s e^{-i\phi_{21}} \end{pmatrix} = -sc\left(1 - e^{-i\phi_{21}}\right)$$

$$= -2i \sin\theta\cos\theta\, e^{-i\phi_{21}/2} \sin\left(\frac{\phi_{21}}{2}\right)$$

$$= -i \sin(2\theta)\, e^{-i\phi_{21}/2} \sin\left(\frac{\phi_{21}}{2}\right)$$

$$P(\nu_e \to \nu_\mu) = |\langle \nu_\mu | \nu_e(t)\rangle|^2 = \sin^2(2\theta)\sin^2\left(\frac{\phi_{21}}{2}\right)$$

with $L = ct$

$$\frac{\Delta\phi_{21}}{2} = \frac{\Delta m_{21}^2 c^2}{4\hbar c E_\nu} L = 1.27 \frac{\Delta m^2[eV^2]\, L[km]}{E_\nu[GeV]}$$

for example $\Delta m^2 = 3\times 10^{-3}\, eV^2$, $E_\nu = 1\,GeV$

max mixing $\sin^2\left(\frac{\phi_{21}}{2}\right) = 1$ $\quad \frac{\phi_{21}}{2} = \frac{\pi}{2}$ @ $L = 400\,km$

<u>ν mass limits</u>

Katrin exp. end-point of tritium β decay

$$m_{\nu_e} < 0.8\, eV \text{ @ } 90\% \text{ CL}$$

$\langle\nu\rangle$ mass from cosmological observations (CMB, baryon acoustic oscillations, ...)

$$\sum m_\nu < 0.09\, eV \text{ @ } 95\% \text{ CL}$$

Only 3 light ($m_\nu < \frac{m_Z}{2} \approx 45\,\text{GeV}$) flavors from Z width $\Gamma_Z \approx 500$ MeV measured from $Z \to$ hadrons resonance

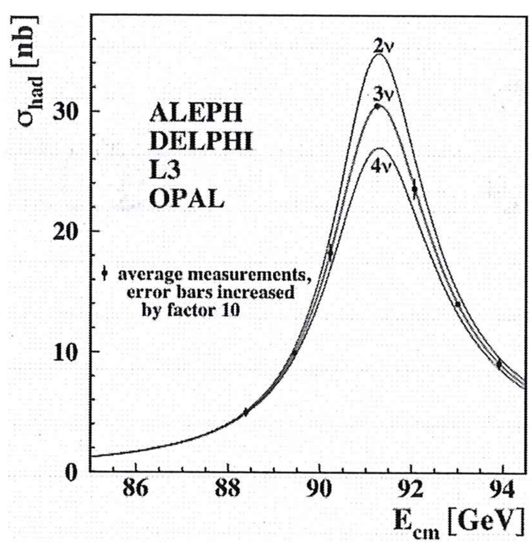


Fig. 1. Measurement of the hadron production cross-section as a function of the LEP centre-of-mass energy around the Z-boson resonance. Combined results from the four LEP experiments are presented. Curves represent the predictions for two, three and four neutrino species. To further convey the high sensitivity of the measurement, uncertainties are magnified tenfold.[14]

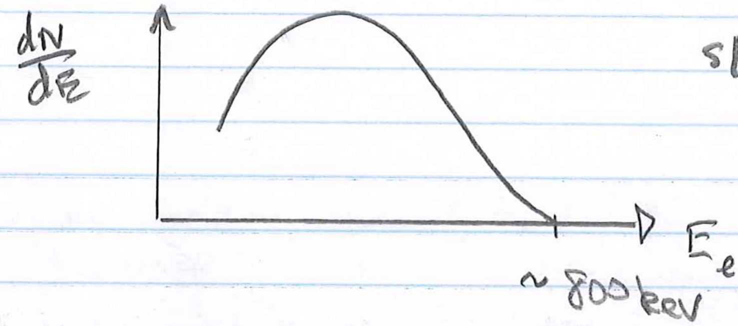
$$\Gamma_Z = \Gamma_{had} + \Gamma_{ee} + \Gamma_{\mu\mu} + \Gamma_{\tau\tau} + N_\nu \Gamma_{\nu\nu}$$

$\underline{N_\nu = 3}$

β decay

for example $n \to p + e^- + \bar{\nu}_e$

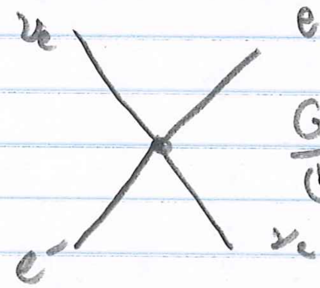
free neutron $\tau \approx 900\,s \approx 14$ minutes



sketch of spectrum

$E, \vec{p}$ conservation $\Rightarrow$ unobserved ν from decay

Fermi contact interaction

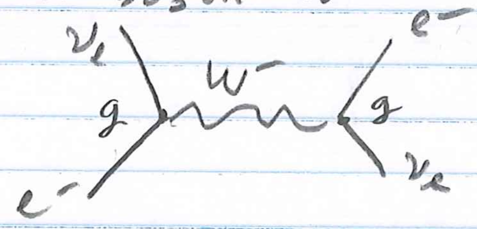


$$\frac{G_F}{(\hbar c)^3} = 1.166 \times 10^{-5}\ \text{GeV}^{-2}$$

$$\sigma = \frac{4}{\pi} G_F^2 (\hbar c k)^2 < \frac{\pi}{2k^2}$$ S-wave unitarity

gives $(\hbar c k)^2 = \frac{\pi^2}{8 G_F}$ $\quad \hbar c k = 300$ GeV limit

Solution is to introduce intermediate vector boson W



V-A law

contact interaction is current current,
decay amplitude (matrix element)

$$\mathcal{M} = \frac{4G}{\sqrt{2}} J^{\mu} J_{\mu}^{+}$$

where current is of the form

$$J_{\ell} = \bar{U}_{\nu}\, \Theta\, U_{\ell} \qquad \ell = \text{lepton } e, \mu, \tau$$

five possible lorentz invariant operators,

$$\Theta = 1,\ \gamma^5,\ \gamma^{\mu},\ \gamma^5\gamma^{\mu},\ \sigma^{\mu\nu}$$

scalar, pseudoscalar, vector
axial-vector, tensor

$$\gamma^5 \equiv i\gamma^0\gamma^1\gamma^2\gamma^3$$

$$\sigma^{\mu\nu} = \frac{i}{2}\left(\gamma^{\mu}\gamma^{\nu} - \gamma^{\nu}\gamma^{\mu}\right)$$

by experiment leptonic current is
pure V-A, parity violating:

$$J_{\ell}^{\mu} = \bar{U}_{\nu}\gamma^{\mu}\frac{1}{2}(1-\gamma^5)\,U_e$$

$$(J_{\ell}^{+})_{\mu} = \bar{U}_e\,\gamma_{\mu}\,\frac{1}{2}(1-\gamma^5)\,U_{\nu}$$

ν left handed, $\bar{\nu}$ right handed

Muon decay $\mu^- \to \nu_\mu + e^- + \bar{\nu}_e$

Putting in W,

$$M = \left(\frac{g}{\sqrt{2}}\right) J_\mu^\lambda \left(\frac{1}{m_W^2 - q^2}\right) \frac{g}{\sqrt{2}} \left(J_e^+\right)_\lambda$$

λ lorentz index

$$\xrightarrow[q^2=0]{} \frac{g^2}{2} \frac{1}{m_W^2} J_\mu^\lambda \left(J_e^+\right)_\lambda$$

then $\boxed{\frac{4G_F}{\sqrt{2}} = \frac{g^2}{2} \frac{1}{m_W^2}}$

from muon lifetime, $E_0 = E_e + E_\nu$

$$\Gamma = \frac{1}{\tau} = \frac{G_F^2 m_\mu^5}{192 \pi^3} = 2.2 \mu s$$

$$\alpha_W = \frac{g}{\hbar c} = \frac{1}{29} > \alpha_{EM}$$

$$m_W = 80 \text{ GeV}$$

Weak force is weak because M_W is heavy

"handedness" chirality

In relativistic (massless) limit (chirality = helicity)

ν left chiral $\quad \overleftarrow{\sigma} \quad \vec{p}$

$\bar{\nu}$ right chiral $\quad \vec{\sigma} \quad \vec{p}$

γ^5 : $\{\gamma^5, \gamma^\mu\} = 0$

$(\gamma^5)^2 = I$; $(\gamma^5)^\dagger = \gamma^5$

in Pauli - Dirac representation

$\gamma^0 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}$ diagonal mass

$\vec{\gamma} = \begin{pmatrix} 0 & \vec{\sigma} \\ -\vec{\sigma} & 0 \end{pmatrix}$ $\gamma^5 = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}$

so $\gamma^\mu \left(\frac{1-\gamma_5}{2} \right) = \left(\frac{1+\gamma_5}{2} \right) \gamma^\mu$

and $\left(\frac{1-\gamma_5}{2} \right)^2 = \frac{1-\gamma_5}{2}$

then $\gamma^\mu \left(\frac{1-\gamma_5}{2} \right) = \left(\frac{1+\gamma_5}{2} \right) \gamma^\mu \left(\frac{1-\gamma_5}{2} \right)$

defining $U_L = \left(\frac{1-\gamma_5}{2} \right) U$

so $\bar{U} = \bar{U} \left(\frac{1+\gamma_5}{2} \right)$

and current is left chiral,

$J_\ell^\mu = \bar{U}_\nu \gamma^\mu \frac{1}{2}(1-\gamma_5) U_\ell$

$= \bar{U}_{\nu_L} \gamma^\mu U_{\ell_L}$

W^- couples to left-chiral current

parity violation => left-right symmetry breaking

Weyl equations

Dirac matricies in Pauli-Dirac representation diagonalizes γ^0 (mass). Weyl representation diagonalizes chirality. more appropriate for weak interactions.

$$\gamma^5 = \begin{pmatrix} -I & 0 \\ 0 & I \end{pmatrix} \qquad \gamma^0 = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}$$

$$\vec{\gamma} = \begin{pmatrix} 0 & \vec{\sigma} \\ -\vec{\sigma} & 0 \end{pmatrix}$$

then Dirac equation with plane wave solution

$$\psi = U(p) e^{ip.x} \quad \text{is}$$

$$\begin{pmatrix} -\vec{\sigma}\cdot\vec{p} & m \\ m & \vec{\sigma}\cdot\vec{p} \end{pmatrix} \begin{pmatrix} \chi \\ \phi \end{pmatrix} = E \begin{pmatrix} \chi \\ \phi \end{pmatrix}$$

we see that mass couples L, R chiral wave functions.

Chiral projection operators,

$$P_L = \tfrac{1}{2}(1-\gamma_5) = \begin{pmatrix} I & 0 \\ 0 & 0 \end{pmatrix}$$

$$P_R = \tfrac{1}{2}(1+\gamma_5) = \begin{pmatrix} 0 & 0 \\ 0 & I \end{pmatrix}$$

$$\psi_L = P_L \psi = \begin{pmatrix} \chi \\ 0 \end{pmatrix} \qquad \psi_R = P_R \psi = \begin{pmatrix} 0 \\ \phi \end{pmatrix}$$

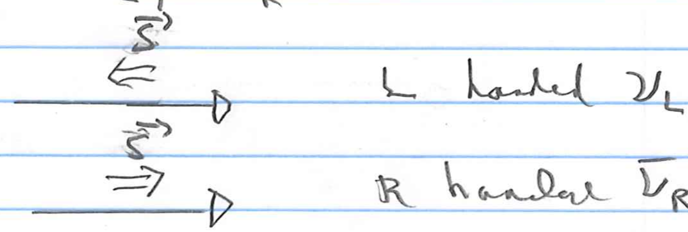
4 comp. ψ is direct sum of spinor representations of Lorentz group.

ultra relativistic limit ($m \approx 0$)

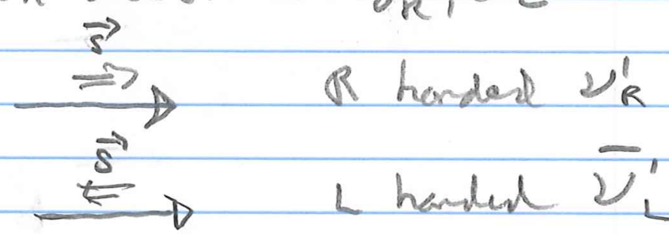
$$-\vec{\sigma}\cdot\vec{p}\,\chi = E\,\chi \qquad E>0$$

$$\vec{\sigma}\cdot\vec{p}\,\phi = E\,\phi \qquad E<0$$

χ describes ν_L, $\bar{\nu}_R$ when



when re-interpreted as $E>0$ solution,
bottom equation describes ν'_R, $\bar{\nu}'_L$



Weak interactions couple to top state (ν) as

left chiral W

Bottom States are sterile with respect to weak interactions W, Z. They would couple to a hypothetical W'

right chiral W'

W' direct search (Atlas @ LHC)

$$M_{W'} \gtrsim 5000 \text{ GeV} \quad (95\% \text{ CL})$$

Returning to Pauli-Dirac rep. to look at chirality,

$$\gamma^5 = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}$$

Dirac spinor $$u^{(s)}(\vec{p}) = \sqrt{E+m} \begin{pmatrix} \chi^s \\ \frac{\vec{\sigma}\cdot\vec{p}}{E+m} \chi^s \end{pmatrix}$$

$$\xrightarrow[E \gg m]{} \sqrt{E} \begin{pmatrix} \chi^s \\ \vec{\sigma}\cdot\hat{p}\, \chi^s \end{pmatrix} \qquad \hat{p} = \vec{p}/|\vec{p}| = \vec{p}/E$$

so

$$\gamma^5 u^{(s)}(\vec{p}) = \sqrt{E} \begin{pmatrix} \vec{\sigma}\cdot\hat{p}\, \chi^s \\ \chi^s \end{pmatrix}$$

$$= \sqrt{E} \begin{pmatrix} \vec{\sigma}\cdot\hat{p}\, \chi^s \\ (\vec{\sigma}\cdot\hat{p})^2 \chi^s \end{pmatrix}$$

since $(\vec{\sigma}\cdot\hat{p})^2 = I$

$$= \begin{pmatrix} \vec{\sigma}\cdot\hat{p} & 0 \\ 0 & \vec{\sigma}\cdot\hat{p} \end{pmatrix} u^{(s)}$$

$$= \vec{\Sigma}\cdot\hat{p}\, u^{(s)}$$

helicity operator. in ultrarelativistic limit,

chirality = helicity

Note in Weyl rep $\vec{\Sigma}\cdot\vec{p}$ is the same

For non-relativistic particles, for example μ^+ produced in weak decay

$$\pi^+ \longrightarrow \mu^+ \nu_\mu$$

π^+ has $S=0$,

$$\nu_\mu \xleftarrow{\vec{S}\,\Rightarrow} \qquad \xrightarrow{\vec{S}\,\Leftarrow} \mu^+$$

to conserve angular momentum.

μ^+ produced as right-chiral from V-A law, probability to be in Left helicity state is.

$$\text{Prob}_L = \tfrac{1}{2}\left(1 - \tfrac{v}{c}\right) \xrightarrow[v/c=1]{} 0$$

and right helicity

$$\text{Prob}_R = \tfrac{1}{2}\left(1 + v/c\right) \xrightarrow[v/c=1]{} 1$$

since angular momentum forces μ^+ to be in Left helicity state, decay is suppressed by factor $(1 - v/c)$.

e^+ in decay $\pi^+ \to e^+ \nu_e$ is more relativistic, so even more suppressed.

decay rate $\quad m_{\pi}^{+} \to l^+ \nu_l \qquad l = e, \mu$

$$\Gamma \propto (\text{phasespace}) \times \left(1 - \frac{v}{c}\right)$$

Relativistic kinematics

$$|\vec{p}_\nu| = |\vec{p}_e| \equiv p$$

$$E_0 = m_\pi = p + \sqrt{p^2+m^2} \Rightarrow p = \frac{m_\pi^2 - m^2}{2m_\pi}$$

phase space $p^2 \frac{dp}{dE_0}$

$$\frac{dE_0}{dp} = 1 + \frac{p}{\sqrt{p^2+m^2}} = 1 + \frac{p}{m_\pi - p} = \frac{m_\pi}{m_\pi - p}$$

$$\frac{dp}{dE_0} = \left(1 - \frac{p}{m_\pi}\right) = 1 - \frac{1}{2m_\pi^2}(m_\pi^2 - m^2)$$

$$= \frac{1}{2}\left(\frac{m_\pi^2 + m^2}{m_\pi^2}\right)$$

then $p^2 \frac{dp}{dE_0} = \frac{1}{4m_\pi^4}(m_\pi^2+m^2)(m_\pi^2-m^2)^2$

$$E^2 = p^2 + m^2 = \frac{(m_\pi^2+m^2)^2}{4m_\pi^2}$$

so $\frac{p}{E} = \frac{v}{c} = \left(\frac{m_\pi^2 - m^2}{m_\pi^2 + m^2}\right)$

$$1 - \frac{v}{c} = \frac{2m^2}{m_\pi^2 + m^2}$$

then decay rate

$$\Gamma \propto p^2 \frac{dp}{dE}\left(1 - \frac{v}{c}\right)$$

$$= \frac{1}{4m_\pi^4}(m_\pi^2+m^2)(m_\pi^2-m^2)^2\left(\frac{2m^2}{m_\pi^2+m^2}\right)$$

$$= \frac{1}{4m_\pi^2} 2m^2(m_\pi^2-m^2)^2$$

$$\boxed{\Gamma \propto \frac{m^2}{2}\left(1 - \frac{m^2}{m_\pi^2}\right)^2}$$

$m_\pi = 135$ MeV, $m_\mu = 106$ MeV, $m_e = 0.5$ MeV

so neglect m_e,

$$\frac{\Gamma(\pi^+ \to e^+ \nu_e)}{\Gamma(\pi^+ \to \mu^+ \nu_\mu)} = \frac{m_e^2}{m_\mu^2} \frac{1}{\left(1 - \frac{m_\mu^2}{m_\pi^2}\right)^2}$$

$$= 1.3 \times 10^{-4}$$

called helicity supression

Majorana ν ?

electron has 4 states

e^- (spin up, down), e^+ (spin up, down)

or in terms of chirality

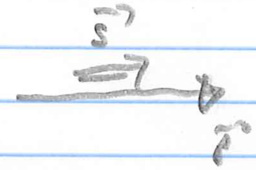
$$P_R + P_L = \tfrac{1}{2}(1+\gamma_5) + \tfrac{1}{2}(1-\gamma_5) = I$$

particle	e^-_L	e^-_R	ν_L	ν_R'
antiparticle (denoted by charge or over-bar)	e^+_R	e^+_L	$\bar{\nu}_R$	$\bar{\nu}_L'$
	electron		neutrino	

as far as we know ν_R', ν_L' do not exist.
(still looking)

then neutrino states are

ν_L



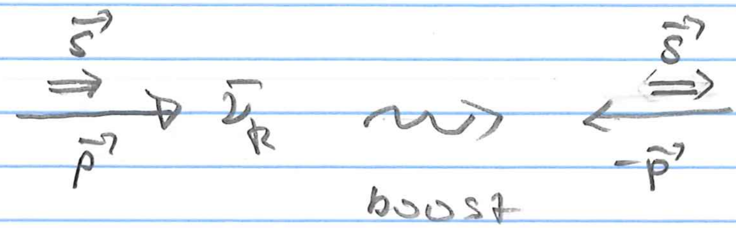
$\bar{\nu}_R$

where all we know is we see

$\ell^- \bar{\nu}_R$, $\ell^+ \nu_L$

right handed anti-neutrino   Left handed neutrino

We can Lorentz boost to
frame where $\vec{p} \to -\vec{p}$, spin does not change



in boosted frame $\bar{\nu}_R$ is just like ν_L.

Could it be the <u>same state</u>? Called <u>Majorana</u> ν.

[Could anti-neutrino $\bar{\nu}_R$ be the same as neutrino ν_L with just opposite helicity?]

Since ν are ultra-relativistic we can never do this experiment. Only way to know is to look for neutrinoless double-β decay.

Some nuclei, e.g. ^{76}Ge are pauli-blocked from beta decay but can undergo <u>double</u> beta decay

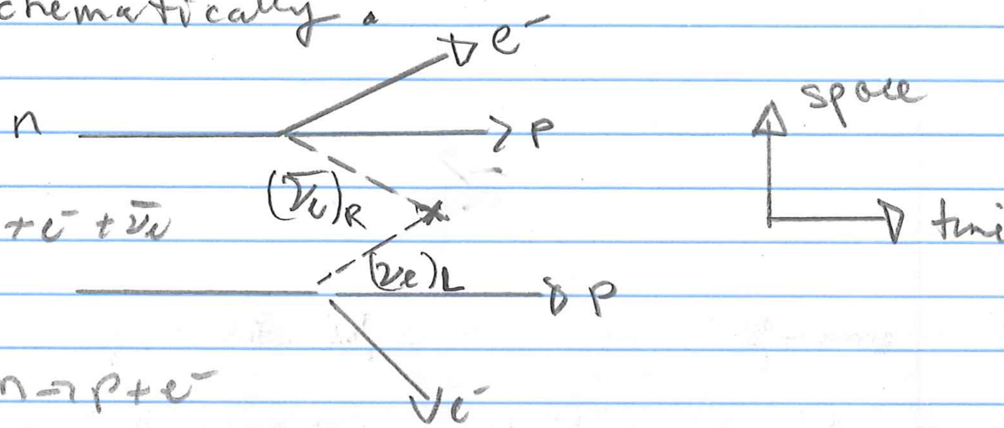
$$^{76}Ge \to {}^{76}Se + e^- \bar{\nu}_e + e^- \bar{\nu}_e$$

germanium → selinium

$$\tau_{1/2} = 2 \times 10^{21} \text{ years.}$$

Since we can think of anti particles as particles moving backwards in time,

schematically:



$n \to p + e^- + \bar{\nu}_e$

$\nu_e + n \to p + e^-$

then $nn \to pp\,e^-e^-$ no neutrinos!

can only happen if $\bar{\nu}_e$, ν_e are particle, antiparticle. (Majorana ν).

Expected half-life $\sim 10^{25}$ years.

If ν is Majorana, then cannot have Dirac mass by interacting with Higgs like other fermions unless

ν' exists or

Some new physics creates Majorana mass terms. Also, Majorana ν introduces 2 new CP phases in mixing matrix. Not observable in oscillation experiments. Possible solution to problem of baryogenesis (matter/antimatter) asymmetry puzzle - universe is all matter.