Phys 522 Spring 2024

A Bit of Nuclear Physics

Fundamental theory quantum chromodynamics (QCD) highly nonlinear theory that cannot be solved to yield potential. Effective theories (approximate models) based on meson exchange yield good description at low (nuclear) energies – strong nuclear force.

meson – strongly interacting boson, carrier of strong-nuclear force.

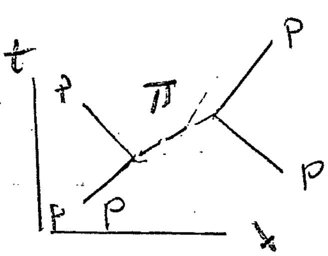
Basic properties of strong-nuclear force:

① Short range

$$V(r) = \frac{g^2}{r} e^{-r(m_\pi c/\hbar)}$$

range, set by π-meson (pion) Compton wavelength

Virtual π exchange allowed by uncertainty principle



Feynman space-time diagram

$$(c\Delta t)(m_\pi c^2) = \hbar c$$

$$\frac{\hbar c}{m_\pi c^2} = \frac{200\ \text{MeV}\cdot\text{fm}}{140\ \text{MeV}} \approx 1\ \text{fm}$$

(2) Strong Coupling

$$\alpha_s = \frac{g^2}{\hbar c} \simeq 15 \gg \alpha_{EM} = \frac{1}{137}$$

expect binding energies $\sim \alpha_s^2 = (2\times10^3)^2 \alpha_{EM}$

order of magnitude $10^6 \times (10\ \text{eV atomic}) = \underline{10\ \text{MeV}}$

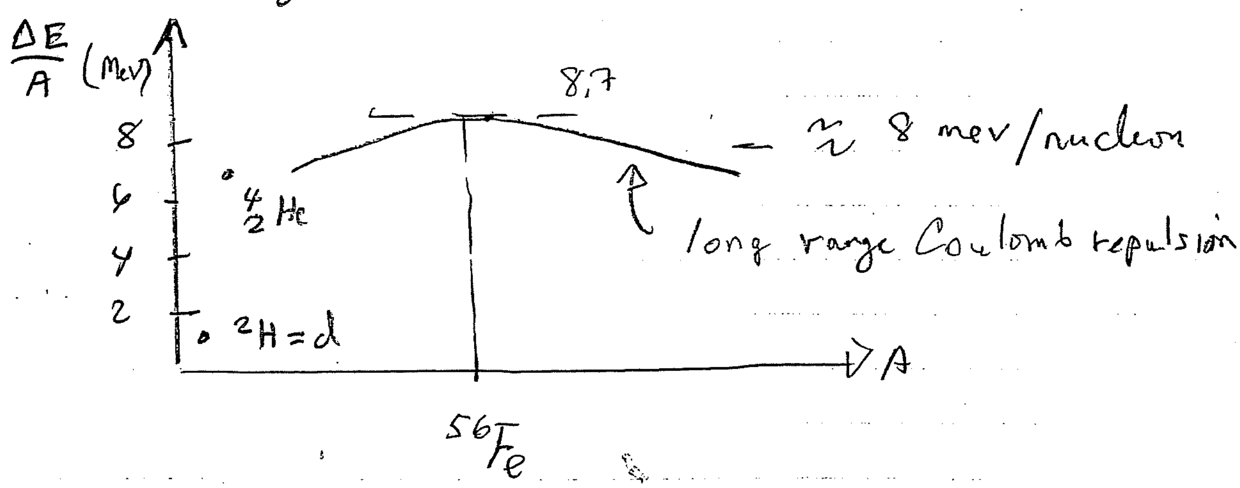
(3) nuclear radius

like sphere packing

$$r_A \simeq (1.2\ \text{fm})\, A^{1/3}$$

nuclear density $\simeq 10^{15}\ \text{g/cm}^3$

(4) Binding energy per nucleon

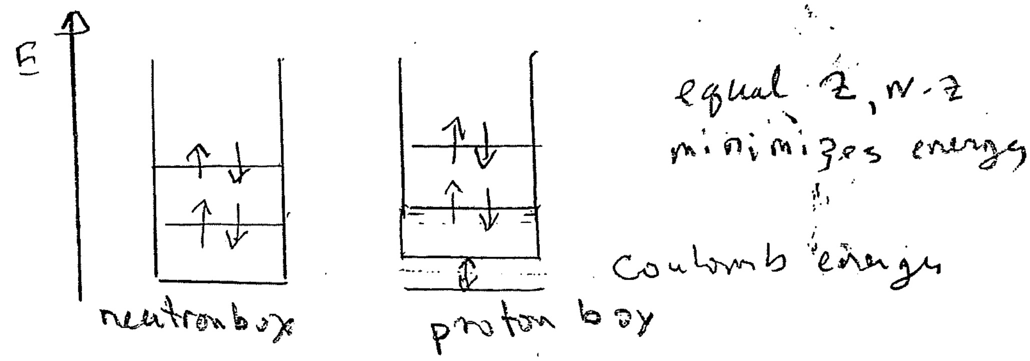


(i) saturation due to short range

(ii) long range Coulomb $\Rightarrow$ more neutrons needed to bind heavy nuclei

(5) Charge independence

light nuclei have $Z \approx (A-Z)$ result of Pauli exclusion principle. Simple box model



Charge independence verified in nn, np, pp scattering

Charge independence formalized by iso-spin symmetry

$$N = \begin{pmatrix} p \\ n \end{pmatrix} \qquad I = \frac{1}{2}$$

$$\pi = \begin{pmatrix} \pi^+ \\ \pi^0 \\ \pi^- \end{pmatrix} \qquad I = 1$$

Write nuclear Hamiltonian in iso-spin invariant form

States must be identical fermion, completely anti-symmetric

$$\psi = (\text{space})(\text{spin})(\text{isospin})$$

deuteron $d = np$ is very weakly bound, measured by photo-disintegration to be $E_b = 2.2$ MeV

$$\gamma + d \to n + p$$

Why is di-neutron (nn) not bound?

Nuclear force is charge independent but Spin dependent.

spin aligned potential greater than spin anti-aligned.

The deuteron has spin 1. By antisymmetry of wave function, isospin = 0. The di-neutron is part of isospin = 1 triplet.
Coulomb force breaks isospin symmetry, so not all states have same energy. PP can be unbound while nn might be bound but spin states are

$$\chi_d = |1, m\rangle \quad \text{spin aligned} \quad \uparrow\uparrow$$

$$\chi_{nn} = |0, 0\rangle \quad \text{spin anti-aligned} \quad \uparrow\downarrow$$

Since deuteron is so weakly bound, anti-aligned potential does not allow binding.

The deuteron has $l=0$ and spin $=1$.
Actually, deuteron wave function has small $l=2$ component giving small qudrupole moment,

$$Q \simeq 0.3\ e\text{-}fm^2$$

A realistic meson-theoretic potential is
(see for example, Roy & Nigam, _Nuclear Physics_, Wiley)

$$V(x) = \frac{1}{3}\left(\frac{g^2}{4\pi}\right) m_\pi c^2\ \vec{\tau}_1 \cdot \vec{\tau}_2 \left\{ \vec{\sigma}_1 \cdot \vec{\sigma}_2 + \hat{S}_{12}\left(1 + \frac{3}{x} + \frac{3}{x^2}\right)\right\} \frac{e^{-x}}{x}$$

where $x = r\, m_\pi c/\hbar$ and
where σ_i - pauli matrices acting on spin state

τ_i - Pauli matrix acting on iso-spin state

Spin tensor operator

$$\hat{S}_{12} = 3\,(\vec{\sigma}_1 \cdot \hat{e}_r)(\vec{\sigma}_2 \cdot \hat{e}_r) - \vec{\sigma}_1 \cdot \vec{\sigma}_2$$

where $\hat{e}_r$ is unit vector of direction connecting nucleons.