# Muon Magnetic Moment

Final Report of the Muon E821 Anomalous Magnetic Moment Measurement at BNL
February 20, 2006

The muon magnetic moment is related to its intrinsic spin by the gyromagnetic ratio $g_\mu$:

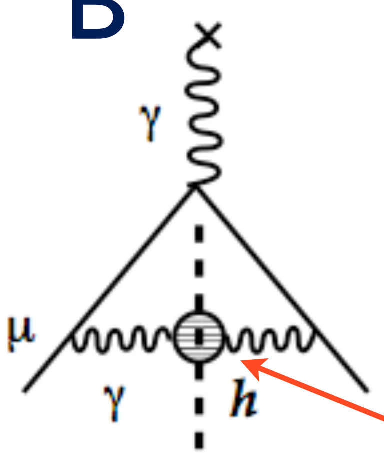
$$\vec{\mu}_\mu = g_\mu \left(\frac{q}{2m}\right) \vec{S}, \tag{1}$$

where $g_\mu = 2$ is expected for a structureless, spin-$\frac{1}{2}$ particle of mass $m$ and charge $q = \pm e$. Radiative corrections (RC), which couple the muon spin to virtual fields, introduce an anomalous magnetic moment defined by

$$a_\mu = \frac{1}{2}(g_\mu - 2). \tag{2}$$

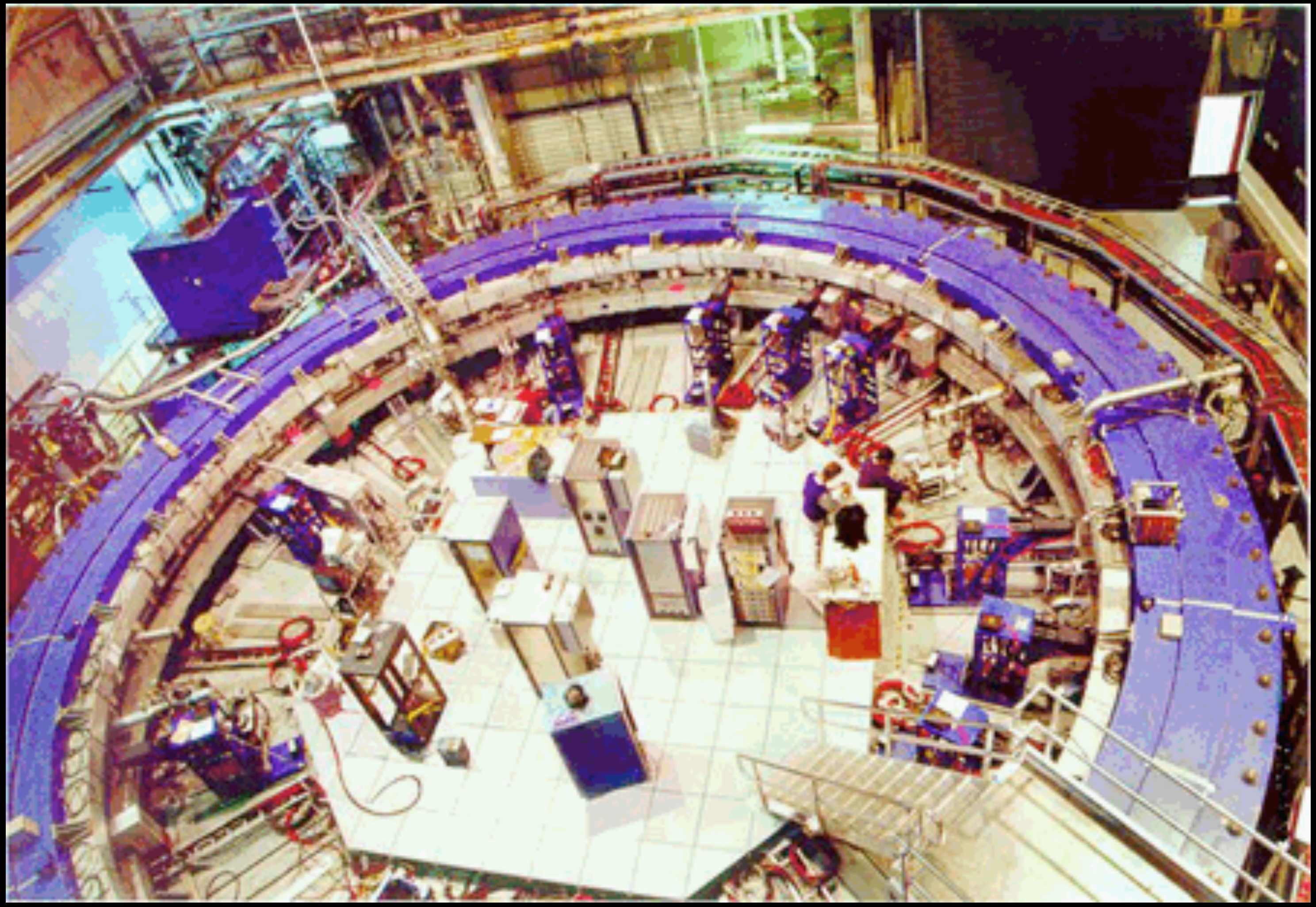
The leading RC is the lowest-order (LO) quantum electrodynamic process involving the exchange of a virtual photon, the "Schwinger term," [1] giving $a_\mu(\text{QED; LO}) = \alpha/2\pi \approx 1.16 \times 10^{-3}$. The complete standard model value of $a_\mu$, currently evaluated to a precision of approximately 0.6 ppm (parts per million), includes this first-order term along with higher-order QED processes, electroweak loops, hadronic vacuum polarization, and other higher-order hadronic loops. The measurement of $a_\mu$, carried out to a similar precision, is the subject of this paper. The difference between experimental and theoretical values for $a_\mu$ is a valuable test of the completeness of the standard model. At sub-ppm precision, such a test explores physics well above the 100 GeV scale for many standard model extensions.

B




(a)

? new stuff ?



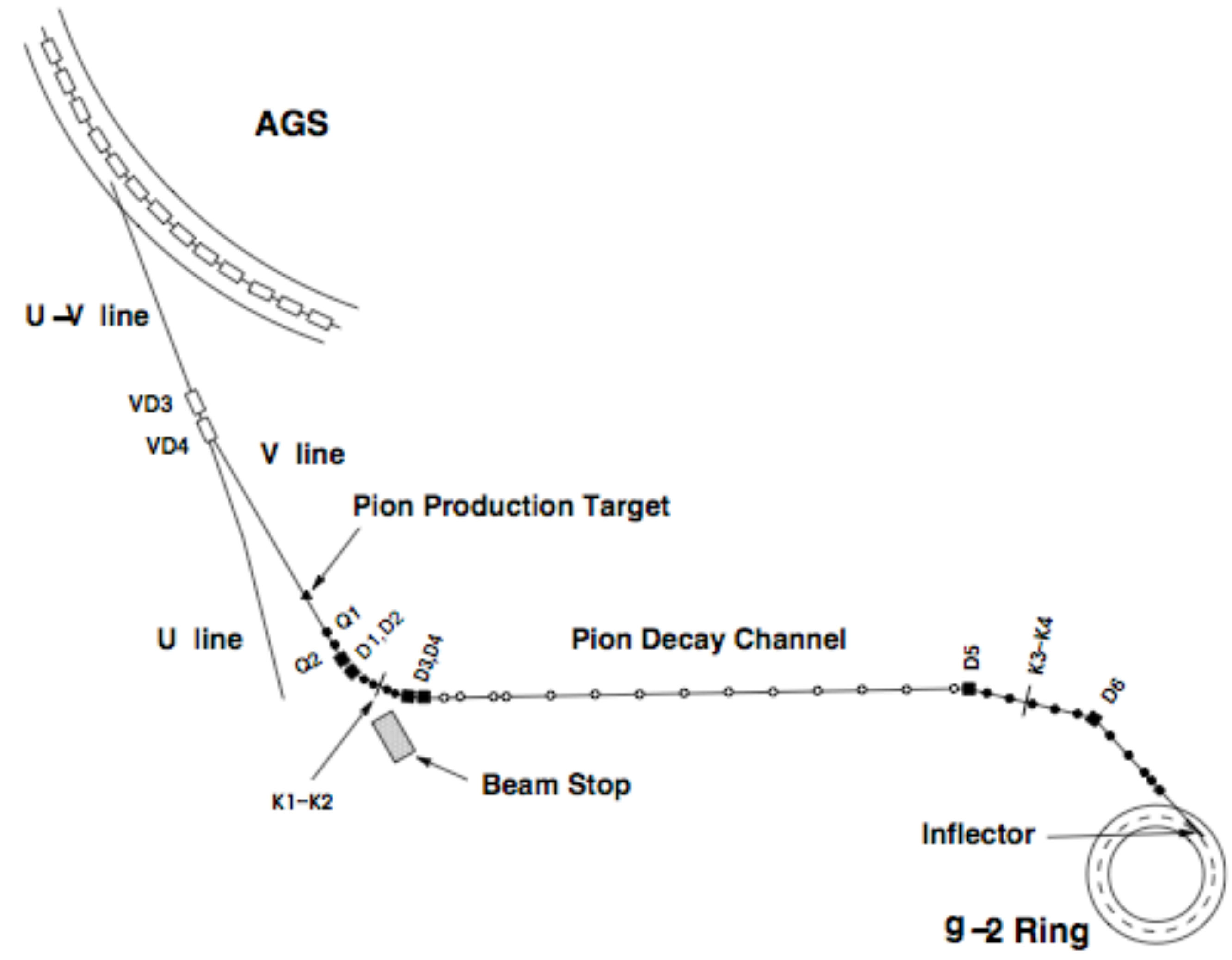



FIG. 3: Plan view of the pion/muon beamline. The pion decay channel is 80 m and the ring diameter is 14.1 m.

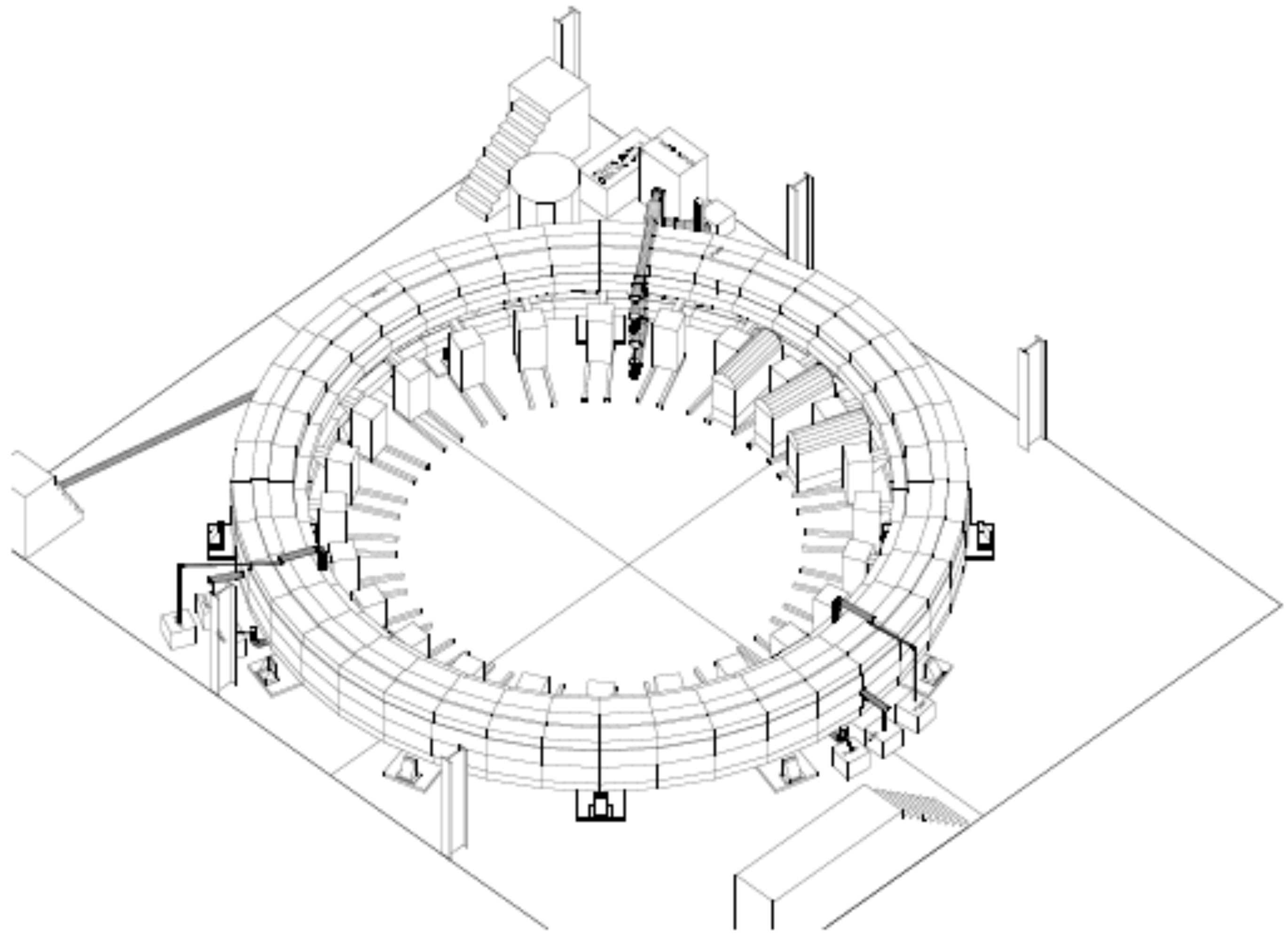
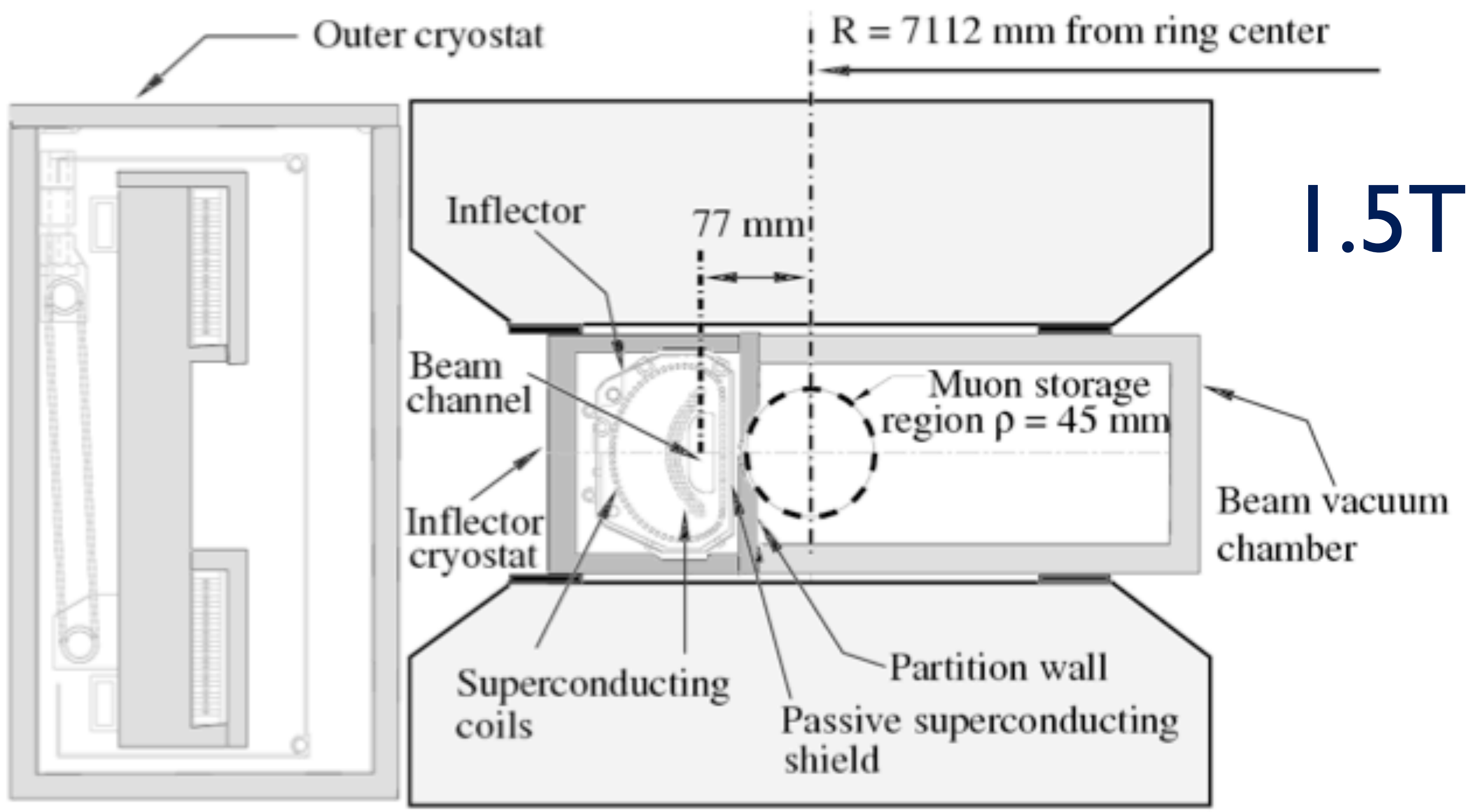


FIG. 6: A 3D engineering rendition of the E821 muon storage ring. Muons enter the back of the storage ring through a field-free channel at approximately 10 o'clock in the figure. The three kicker modulators at approximately 2 o'clock provide the short current pulse, which gives the muon bunch a transverse 10 mrad kick. The regularly spaced boxes on rails represent the electron detector systems.




1.5T

FIG. 4: The inflector/storage ring geometry. The downstream end of the inflector is shown, with the beam channel to the left of the storage region (larger radius). The ring center is to the right. Note the limited space between the pole pieces, which has to contain the inflector and its cryostat along with the beam vacuum chamber. The current in the inflector flows into the page in the "C" shaped arrangement of conductors just to the left of the beam channel, and out of the page in the conductors that form a backward "D". The superconductor crosses over the beam channel to connect the two coils.

# anomolous precession frequency

The cyclotron $\omega_c$ and spin precession $\omega_s$ frequencies for a muon moving in the horizontal plane of a magnetic storage ring are given by:

$$\vec{\omega}_c = -\frac{q\vec{B}}{m\gamma}, \qquad \vec{\omega}_s = -\frac{gq\vec{B}}{2m} - (1-\gamma)\frac{q\vec{B}}{\gamma m}. \tag{3}$$

The anomalous precession frequency $\omega_a$ is determined from the difference

$$\vec{\omega}_a = \vec{\omega}_s - \vec{\omega}_c = -\left(\frac{g-2}{2}\right)\frac{q\vec{B}}{m} = -a_\mu \frac{q\vec{B}}{m}. \tag{4}$$

vertical focusing electric quadrupoles gives a B field in the muon rest frame

$$\vec{\omega}_a = -\frac{q}{m}\left[a_\mu \vec{B} - \left(a_\mu - \frac{1}{\gamma^2 - 1}\right)\frac{\vec{\beta}\times\vec{E}}{c}\right].$$

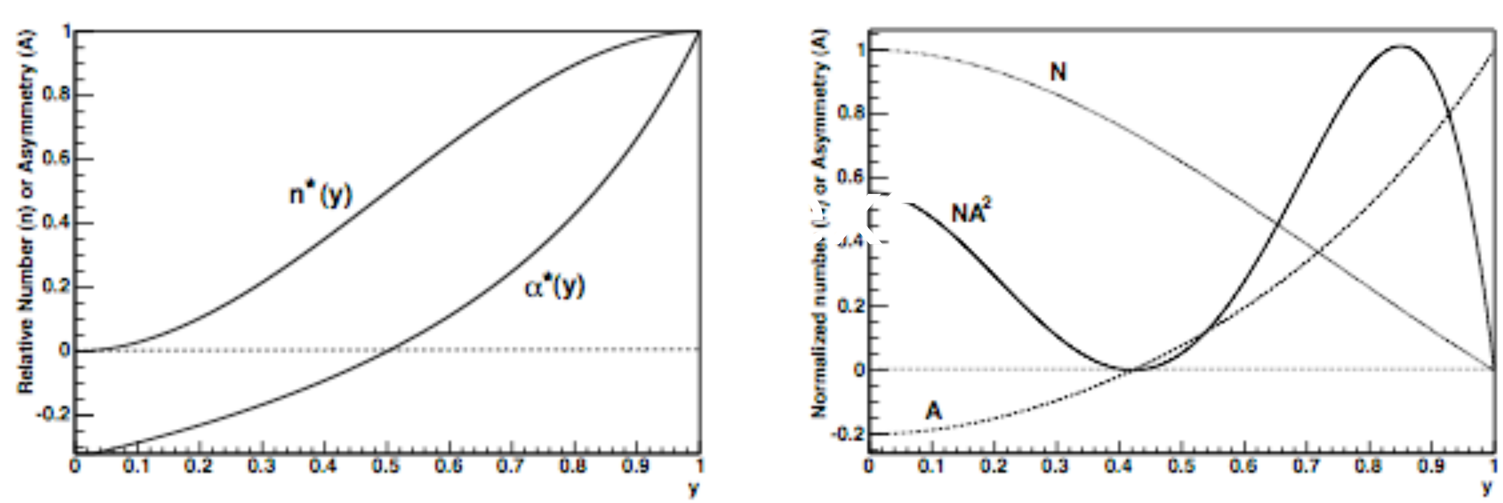
Choose "magic momentum" p=3.094 Gev/c, ϒ=29.3 cancels E term to leading order in QED $\alpha/(2\pi)=1.16\mathrm{x}10^{-3}$

Precision measurement of precession and B

$\mu^- \rightarrow \nu_\mu + e^- + \nu_e$

# Parity-violating weak decay of muon gives correlation between muon spin and electron direction

e opp muon spin




(a) Center-of-mass frame

(b) Lab frame

FIG. 1: Relative number and asymmetry distributions versus electron fractional energy $y$ in the muon rest frame (left panel) and in the laboratory frame (right panel). The differential figure-of-merit product $NA^2$ in the laboratory frame illustrates the importance of the higher-energy electrons in reducing the measurement statistical uncertainty.

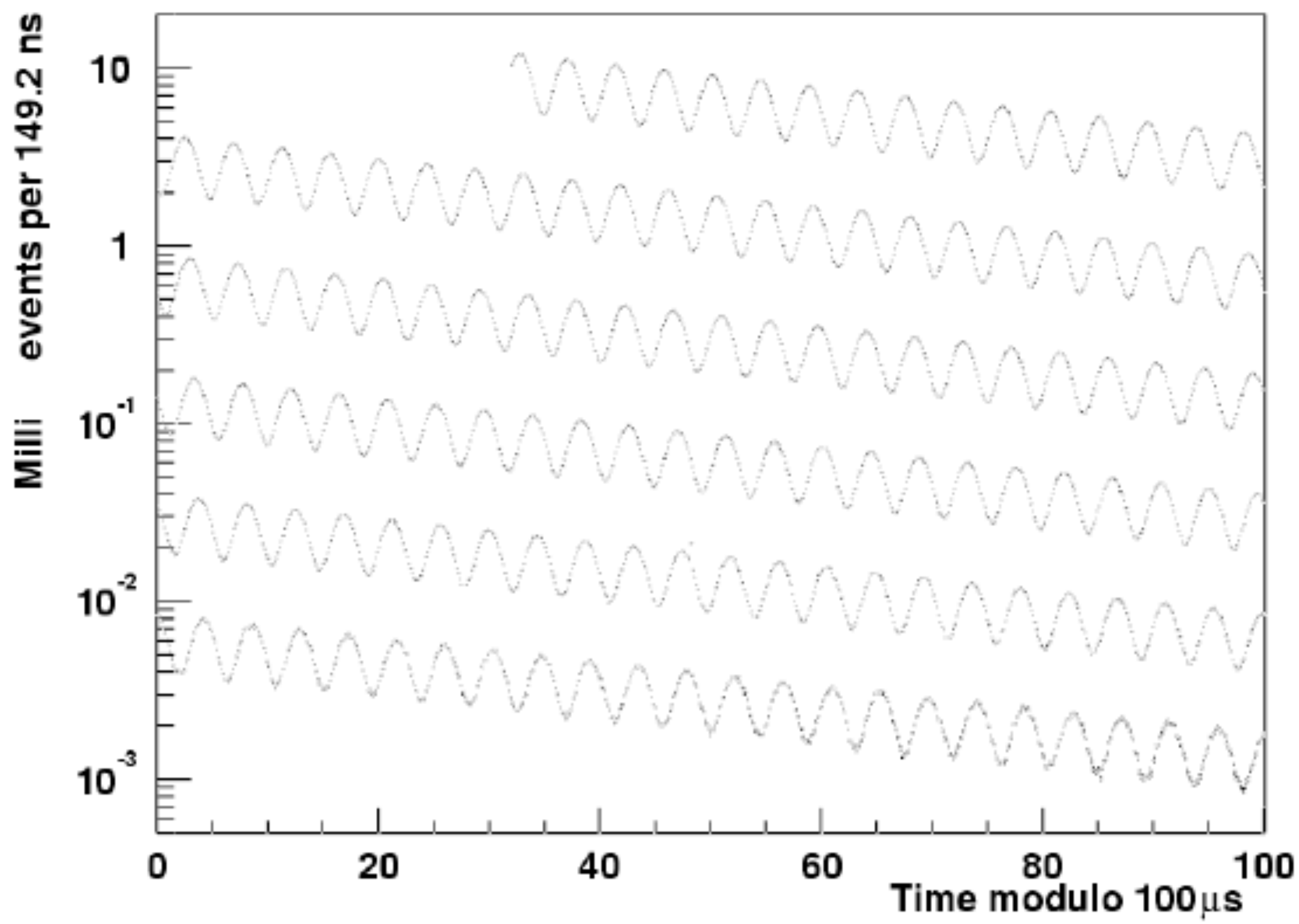



FIG. 2: Distribution of electron counts versus time for the 3.6 billion muon decays in the R01 $\mu^-$ data-taking period. The data is wrapped around modulo 100 $\mu$s.

# B field measurement

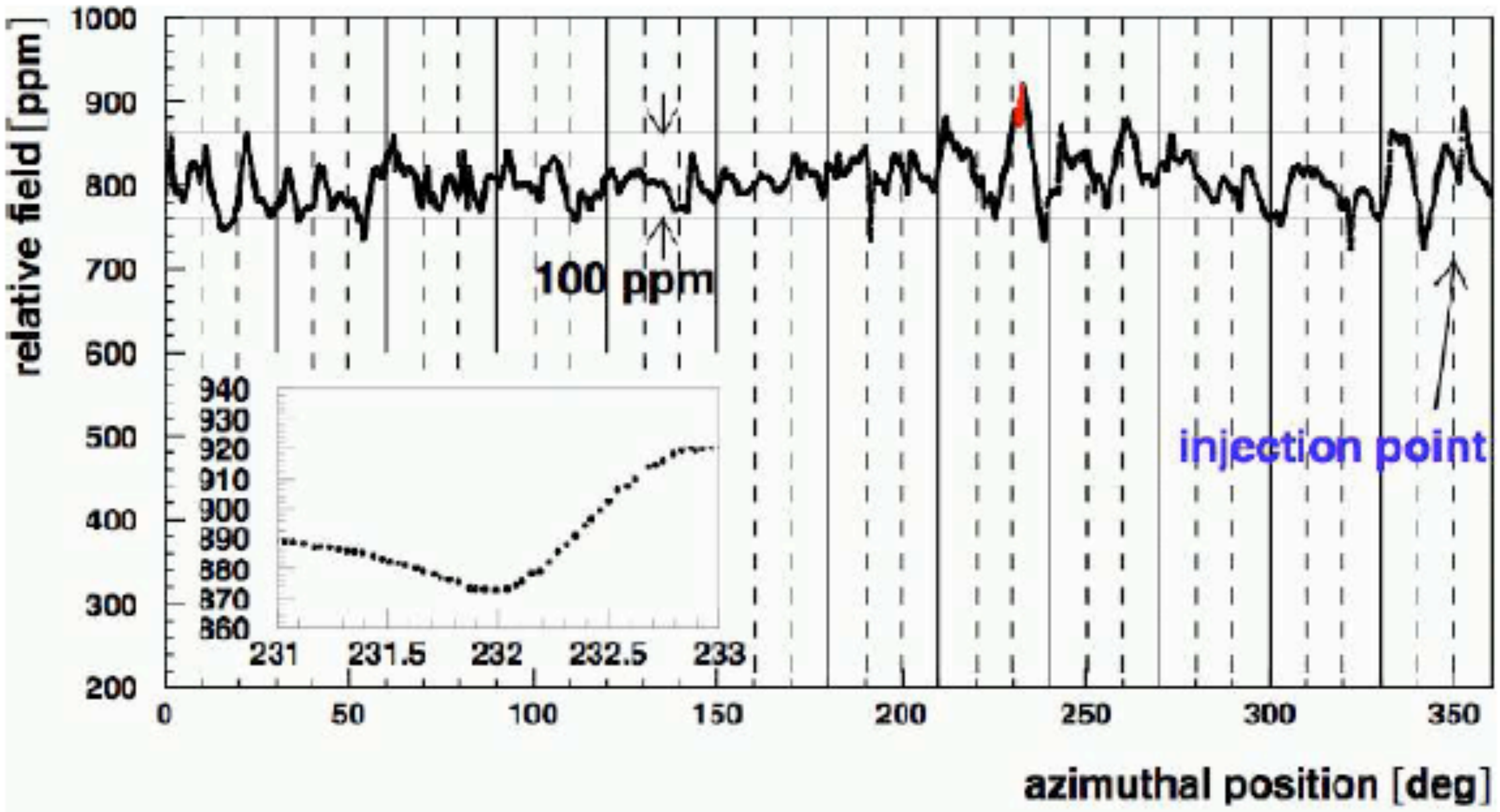



FIG. 26: The NMR frequency measured with the center trolley probe relative to a 61.74 MHz reference versus the azimuthal position in the storage ring for one of the measurements with the field trolley during the R01 period. The continuous vertical lines mark the boundaries of the 12 yoke pieces of the storage ring. The dashed vertical lines indicate the boundaries of the pole pieces. The inset focuses in on the measurements over an interval of two degrees. The point-to-point scatter in the measurements is seen to be small.

To determine $a_\mu$, we divide $\omega_a$ by $\widetilde{\omega}_p$, where $\widetilde{\omega}_p$ is the measure of the average magnetic field seen by the muons. The magnetic field, measured using NMR, is proportional to the free proton precession frequency, $\omega_p$. The muon anomaly is given by:

$$a_\mu = \frac{\omega_a}{\omega_L - \omega_a} = \frac{\omega_a/\widetilde{\omega}_p}{\omega_L/\widetilde{\omega}_p - \omega_a/\widetilde{\omega}_p} = \frac{\mathcal{R}}{\lambda - \mathcal{R}}, \quad (11)$$

where $\omega_L$ is the Larmor precession frequency of the muon. The ratio $\mathcal{R} = \omega_a/\widetilde{\omega}_p$ is measured in our experiment and the muon-to-proton magnetic moment ratio

g_p=5.585 694 713 (046)
m_mu = 106 MeV

$$\lambda = \omega_L/\omega_p = 3.18334539(10) \quad (12)$$

is determined from muonium hyperfine level structure measurements [12, 13].

The standard model theoretical summary is given in Table XVI. Two results are presented, representing the two slightly different $e^+e^-$-based evaluations of the leading-order hadronic vacuum polarization contribution. The theoretical expectation should be compared to our experimental result (Eq. 58):

$$a_\mu(\text{Expt}) = 11\,659\,208.0(6.3) \times 10^{-10} \quad (0.54\,\text{ppm}).$$

The difference

$$\Delta a_\mu(\text{Expt} - \text{SM}) = (22.4 \pm 10 \text{ to } 26.1 \pm 9.4) \times 10^{-10}, \tag{64}$$

has a significance of 2.2 to 2.7 standard deviations. Use of the $\tau$-data gives a smaller discrepancy.

To show the sensitivity of the measured muon $(g-2)$ value to the electroweak gauge bosons, the electroweak contribution given in Eq. 61 is subtracted from the standard model values in Table XVI. The resulting difference with theory is

$$\Delta a_\mu = (38 \text{ to } 41 \pm 10) \times 10^{-10}, \tag{65}$$