

**Recitation #2**  
**Quantum 522**

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1. Consider a classical orbit of the electron around the proton. Use the Bohr quantization rule  $mvr = n\hbar$  to get an estimate of the radius of hydrogen. Next, use the virial theorem  $\langle T \rangle = -\frac{1}{2}\langle V \rangle$  to show  $\langle 1/r \rangle = 1/(a_0 n^2)$ . Use Kramer's relation with  $s = 1$  to get the  $\langle r \rangle$ ,  $\frac{2}{n^2}\langle r \rangle - 3a_0 + (\ell(\ell + 1) a_0^2 \langle 1/r \rangle = 0$
2. The  $H_\alpha$  line corresponds to the transition from the  $n = 3$  to  $n = 2$  energy levels in hydrogen. Calculate the fractional splitting of this line due to deuterium. ( $m_p/m_e = 1836$ .)