Quantum 522: TEST # 1

Please return the test with your work. No book, no notes, no google, no discussion. Calculator OK.

#1) Calculate the energy correction for the $n = 2$ states of hydrogen in the weak field Zeeman effect. The needed Clebsch-Gordon coefficients are:

$$|j = \ell \pm \frac{1}{2}, m\rangle = \pm\sqrt{\frac{\ell \pm m + \frac{1}{2}}{2\ell + 1}}\,|m_\ell = m - \frac{1}{2}, m_s = \frac{1}{2}\rangle$$
$$+\sqrt{\frac{\ell \mp m + \frac{1}{2}}{2\ell + 1}}\,|m_\ell = m + \frac{1}{2}, m_s = -\frac{1}{2}\rangle$$

#2) Consider a spherically symmetric potential $V(r)$ that vanishes at infinity and for which there is at least one bound state. Use the variational method to show that the lowest bound state has no nodes and is therefore non-degenerate. You may assume that the lowest- bound-state wave function is spherically symmetric.

#3) Consider an isotropic harmonic oscillator in two dimensions. The Hamiltonian is given by

$$H_0 = \frac{P_x^2}{2m} + \frac{P_y^2}{2m} + \frac{m\omega^2}{2}\left(x^2 + y^2\right)$$

a. What are the energies of the three lowest-lying states? Is there any degeneracy?

b. We now apply a perturbation

$$V = \delta m\omega^2 xy$$

where δ is a dimensionless real number much smaller than unity. Find the zeroth-order energy eigenket and the corresponding energy to first order [that is, the unperturbed energy obtained in (a) plus the first-order energy shift] for each of the three lowest-lying states.

c. Solve the $H_0 + V$ problem exactly. Compare with the perturbation results obtained in (b).

Recall: $a = \sqrt{\frac{m\omega}{2\hbar}}\left(x + \frac{i}{m\omega}p\right)$