Physics 301

## Legendre Transformation

- The tangent to the curve U(S, V) at the point S is the function  $T(S) = \left(\frac{\partial U}{\partial S}\right)_V$ . Notice that as you vary S the slope varies. Show geometrically on the figure that the relation ST = U - F holds.
- The function S(T) can be inverted to give T(S). From dU = TdS PdV and the relation ST = U F prove that F(T, V).
- Prove that  $S(T) = -\left(\frac{\partial F}{\partial T}\right)_V$



Figure 1: Geometric construction for Legendre transform. The curve is drawn for fixed V.