Physics 301: Problems #10
Nov. 1, 2019

1. The nucleus of the nitrogen isotope ^{14}N acts like a spinning oblate sphere of positive charge and therefore has an electric quadrupole moment. The nuclear spin is $1\hbar$ and so in an electric field has 3 energy states with energies $+\epsilon$ for aligned ("up"), 0 for perpendicular to the field, and $+\epsilon$ for anti-aligned ("down") .

   - In thermal equilibrium at temperature T ($\beta \equiv 1/kT$), what is the probability to find the nucleus with spin up?
   - What is the mean energy? What is the heat capacity? Sketch both as a function of T.

2. For a spatial localized system of non-interacting particles, the partition function for N of them is $Z = (Z_1)^N$ where Z_1 is the partition function of a single particle at a single site. There is no $N!$ combinatorial factor as they are distinguished by location.

   - Confirm this formula for a two-state paramagnet with two molecules in a magnetic field, with aligned/anti-aligned energies $\mp\epsilon$ by writing the 4 distinct states and writing the partition function as a sum over all the energy states.
   - Find the mean energy for a system of N paramagnets,

   $$\langle E \rangle = -\frac{\partial \ln \mathrm{Z}}{\partial \beta}$$

   - Use the result of HW 6.18 "plus"

   $$(\Delta E)^2 = -\frac{\partial \langle E \rangle}{\partial \beta}$$

   to calculate the fractional uncertainty in energy $\Delta E / |\partial \langle E \rangle|$.