Physics 301: Problems #9
Oct. 25, 2019

From Baierlein 12.11

1. We defined
$$C_p = \frac{\Delta U - (-p\Delta V)}{\Delta T} = \left(\frac{\partial U}{\partial T}\right)_p + p\left(\frac{\partial V}{\partial T}\right)_p$$
Use the fact that $U(T, V, N)$ and $V(p, T)$ from the equation of state (EOS) to prove that
$$C_p = C_V + \left[\left(\frac{\partial U}{\partial V}\right)_T + p\right]\left(\frac{\partial V}{\partial T}\right)_p$$
Pay careful attention to what is held fixed in the partial derivatives.

2. For a monatomic ideal gas, show that $C_p/C_v = 1 + 2/3$

3. For the Van der Waal gas Baierlein tells us that the log of the partition function is
$$\ln Z_{VdW} = \ln\left[\frac{(V - Nb)^N)}{\ell_Q^{3N} N!}\right] + \frac{aN^2}{VkT}$$
where the thermal de Broglie wavelength $\ell_Q = \left(\frac{h^2}{2\pi mkT}\right)^{1/2}$, h is Planck's constant and m is the molecular mass.
Find the EOS from
$$p = kT\left(\frac{\partial \ln Z_{VdW}}{\partial V}\right)$$

4. Find the energy U from
$$U = kT^2\left(\frac{\partial \ln Z_{VdW}}{\partial T}\right)$$

5. Show that for VdW
$$C_p = C_v + \frac{Nk}{1 - A}$$
where
$$A = (1 - Nb/V)^2 \frac{2aN}{kTV}$$
Therefore $(C_p/C_V)_{VdW} > 1 + 2/3$.