Physics 304 Spring 2019

Lecture 17: Continuous media Ch 16

mechanics of deformable media (e.g. solids, fluids)

<u>continuum hypothesis</u> – relevant scales contain so many molecules (Avogadro 6×10^{23} molecules/mole) that matter is approximately continuous.

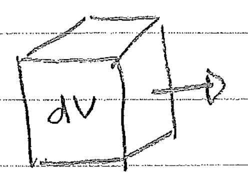
<u>Two types of forces</u>:

① Volume $\vec{F} \propto$ volume

e.g. gravity $d\vec{F} = \rho(\vec{r})\,\vec{g}\,dV$

② Surface $\vec{F} \propto$ area

e.g. hydrostatic pressure $d\vec{F} = -p\,d\vec{A}$

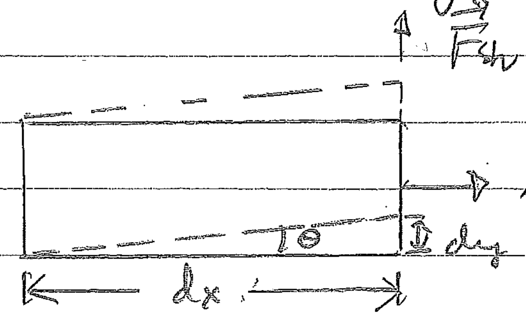


$d\vec{A}$ points out of volume

$$\left.\begin{array}{ll} d\vec{F}_{hydrostatic} & \text{points inward} \\ d\vec{F}_{tension} & \text{points outward} \\ d\vec{F}_{shear} & \text{points tangential} \end{array}\right\} \text{surface forces}$$

Shear force:

showing a cross section of a Volume –



solid – before shear

dotted – deformation under shear $\vec{F}_{sh}$

Deformable solid

We consider only small, elastic deformations.

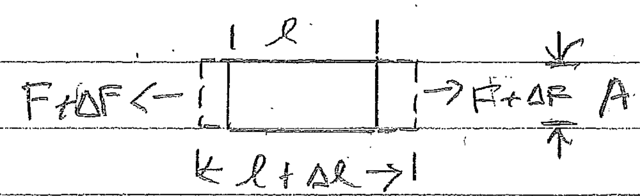
simplest example is Hooke's law

$$F = k\,\Delta x$$

"stress" α "strain"

k depends on material under stress,

example: wire under tension



solid under force F

dotted, under force $F+\Delta F$

Stress on wire is $\frac{F}{A}$

strain is $\Delta l / l$

$$\frac{\text{Stress}}{\text{strain}} = \frac{\Delta F/A}{\Delta l/l} \equiv M_Y \quad \text{Young's modulus}$$

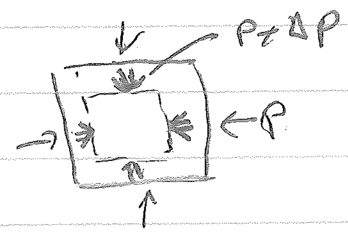
$$[M_Y] = \frac{\text{force}}{\text{area}} = \text{pressure} \quad (\text{Pascal Pa})$$

$$Pa = \frac{N}{m^2} \; ; \quad 1\,\text{ATM} \cong 100\,\text{kPa}$$

<u>note</u> $\Delta F = \underbrace{\left(\frac{A}{l} M_Y\right)}_{\text{force constant "k"}} \Delta l$

<u>Hydrostatic Pressure</u>

cross section of cubic volume $\Rightarrow$



compressed as $P \to P + \Delta P$ $\quad \frac{\Delta V}{V}$ negative

$$\frac{\text{Stress}}{\text{strain}} = \frac{\Delta P}{-\Delta V/V} \equiv M_B \quad \text{bulk modulus}$$

<u>Shear Modulus</u> (see P.2 drawing)

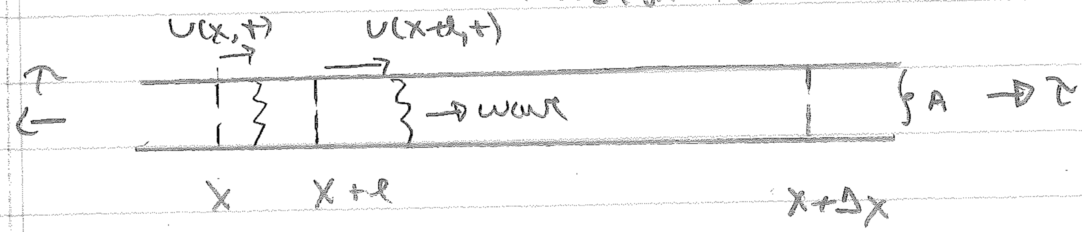
$$\frac{\text{Stress}}{\text{strain}} = \frac{\Delta F/A}{dy/dx} = M_S$$

data table 16.26

	B	S	Y	ρ [g/cm³]
	[GPa]			
iron	90	40	100	7.8
steel	140	80	200	7.8
water	2.2	0	0	1.0

≈ 0 viscosity (inviscid) fluid

Example p. 16.17 longitudinal wave on wire under tension τ



displacement from equilibrium $U(x,t)$.
as pressure longitudinal wave comes along $x \to x + U(x,t)$

consider infinitessimal segment length l.

$x \to x + U(x,t)$

$x + l \to x + l + U(x+l, t)$

subtract to get Δl:

$$\Delta l = U(x+l,t) - U(x,t) = \left.\frac{\partial U}{\partial x}\right|_{x,t} l \equiv U_x(x,t)\, l$$

so $\dfrac{\Delta l}{l} = U_x(x,t)$

Definition of Young's modulus M_Y

$$F(x,t) = A M_Y \left(\frac{\Delta \ell}{\ell}\right) = A M_Y U_x(x,t)$$

net force across Δx is

$$F(x+\Delta x,t) - F(x,t) = A M_Y \left[\partial U_x(x+\Delta x, t) - U_x(x,t) \right]$$

$$= A M_Y U_{xx} \Delta x$$

let ρ be mass/volume of wire

mass of segment $\Delta x = \rho A \Delta x$

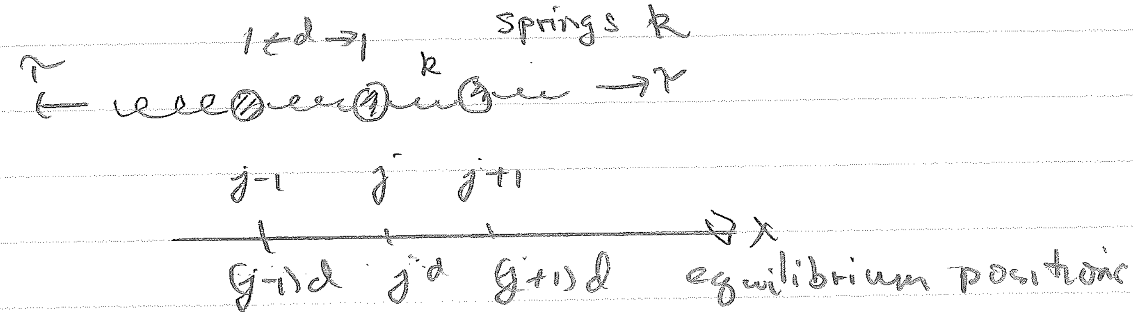
$$\frac{\partial^2 U}{\partial t^2}(x,t)$$

$$F(x+\Delta x,t) - F(x,t) = A M_Y U_{xx} \Delta x = \rho A \Delta x \, U_{tt}(x,t)$$

giving $U_{tt} = \left(\frac{M_Y}{\rho}\right) U_{xx}$

speed of longitudinal wave $c_\ell = \sqrt{\frac{M_Y}{\rho}}$

Instructive to consider loaded string with longitudinal displacements



displacement from equilibrium $U_j(t) = x_j(t) - jd$

$$F_j = k(x_{j+1} - x_j) - k(x_j - x_{j-1})$$

$$= k\underbrace{(U_{j+1} - U_j)}_{\approx \frac{\partial U_{j+1}}{\partial x} d} - k\underbrace{(U_j - U_{j-1})}_{\approx \frac{\partial U_j}{\partial x} d}$$

$$= k\frac{\partial U_{j+1}}{\partial x} d - k\frac{\partial U_j}{\partial x} d \approx kd^2 \frac{\partial^2 U_j}{\partial x^2}$$

with $F_j = m\dfrac{\partial^2 U_j}{\partial t^2} = kd^2 \dfrac{\partial^2 U_j}{\partial x^2}$

in limit $jd \xrightarrow[\substack{d \to 0 \\ N \to \infty}]{} x$ and $\dfrac{m}{d} = \rho A$

$$\rho A d \frac{\partial^2 u}{\partial t^2} = kd^2 \frac{\partial^2 u}{\partial x^2}$$

with $M_y = kdA$ we have

$$\boxed{\frac{\partial^2 u}{\partial t^2} = \frac{M_y}{\rho}\frac{\partial^2 u}{\partial x^2}}$$