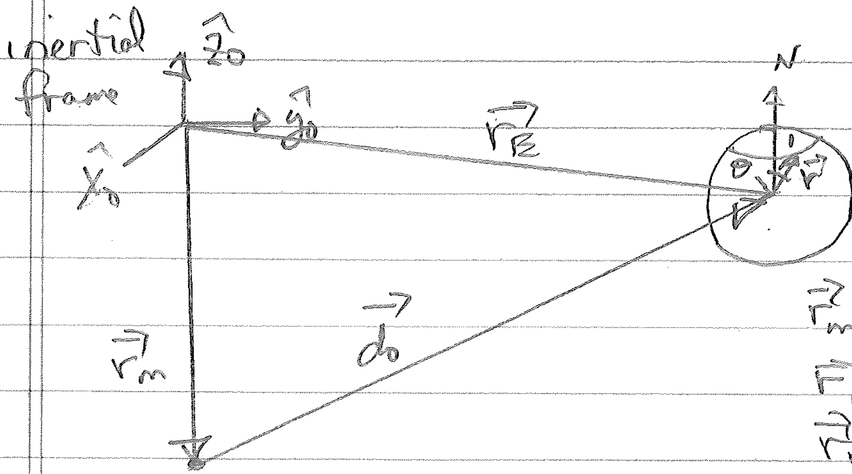
Physics 304 Spring 2015

Lecture 6: Tides (Chapter 9)

Gravity $1/r^2$ force law implies field gradient:



θ is polar angle "co-latitude"

$\vec{r}_m$ moon
$\vec{r}_E$ earth
$\vec{r}_1$ point on E surface

inertial frame

not drawn

relative to earth $\vec{r} = \vec{r}_1 - \vec{r}_E$

relative to moon $\vec{d} = \vec{r}_1 - \vec{r}_m = \vec{r} + \vec{d}_0$

for mass m at point 1, gravity due to Earth & moon,

$$m\ddot{\vec{r}}_1 = -\frac{G M_E m \hat{r}}{r^2} - \frac{G M_m m}{d^2}\hat{d}$$

force on E due to moon

$$M_E \ddot{\vec{r}}_E = -\frac{G M_m M_E}{d_0^2}\hat{d}_0$$

Earth-Moon CM 0.74 R_E

Acceleration difference at Earth surface, subtract + divide by m

$$\ddot{\vec{r}}_1 - \ddot{\vec{r}}_E = \ddot{\vec{r}} = \underbrace{-\frac{GM_E}{r^2}\hat{r}}_{\vec{g}} - GM_m\left[\frac{\hat{d}}{d^2} - \underbrace{\frac{\hat{d}_0}{d_0^2}}_{=\vec{A}}\right]$$

$\vec{A} = -\frac{GM_m\hat{d}_0}{d_0^2}$ acceleration of E - cm frame

$\vec{g}$ is canceled by surface normal force $\vec{F}_N$ (e.g. bouyancy) not included so far.

Leaves tidal force $\vec{F}_{Tide}$,

$$\ddot{\vec{r}} = -GM_M\left(\frac{\hat{d}}{d^2} - \frac{\hat{d}_0}{d_0^2}\right) = \frac{\vec{F}_{tide}}{m}$$

Moon is very far away relative to R_E ($R_E \ll d_0$)

$R_E = 6\times10^3$ km

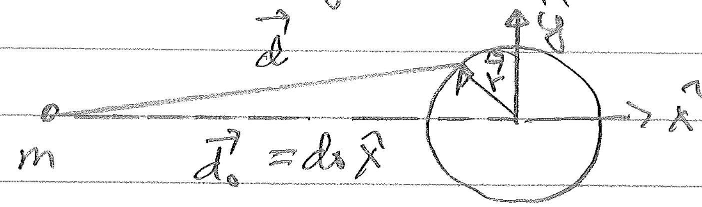
$d_0 = 384\times10^3$ km

$R_E/d_0 = 0.016 = \varepsilon = 1/64$

Easier to expand potential

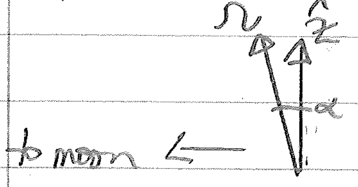
Tidal Potential

Coordinate system at center of earth in E-m plane



$\vec{r} = x\hat{x} + y\hat{y}$

Note, α is tilt of E-axis w.r.t. E-m orbital plane (tilted $\sim 5^\circ$ w.r.t. E-sun plane, ecliptic)



$\alpha = 17^\circ \sim 29^\circ$ as moon's elliptical orbit precesses slowly about E-sun orbital plane (ecliptic)

$$\vec{d} = \hat{x} d_0 + \vec{r}$$

$$\frac{1}{m} F_{tid} = -\vec{\nabla}_{\vec{r}}\, U(r) \qquad \vec{\nabla}_r = \hat{x}\frac{\partial}{\partial x} + \hat{y}\frac{\partial}{\partial y}$$

2^{nd} term in force is easy

$$\frac{\hat{d_0}}{d_0^2} = -\vec{\nabla}\left(\frac{-x}{d_0^2}\right)$$

First term $\dfrac{\hat{d}}{d^2} = -\vec{\nabla}_r\,(?)$ $\qquad \vec{d} = \vec{d_0} + \vec{r}$

$$d^2 = d_0^2 + r^2 + 2\,\vec{r}\cdot\hat{x} d_0 = d_0^2 + r^2 + 2x d_0$$

$$\frac{\partial d^2}{\partial x} = 2x + 2d_0 \quad ; \quad \frac{\partial d^2}{\partial y} = 2y$$

$$\vec{\nabla}(d^2) = 2(\vec{r} + \vec{d_0})$$

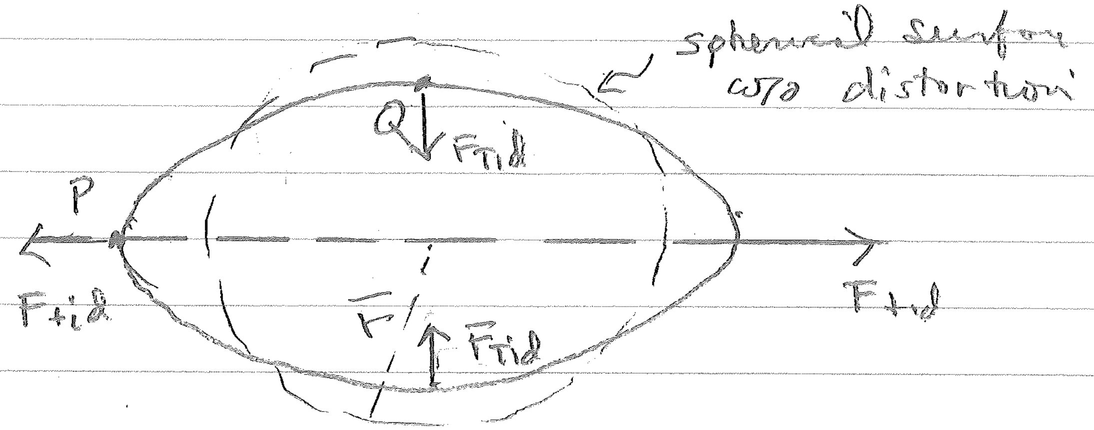
$$-\vec{\nabla}_r\left(\frac{1}{d}\right) = \frac{1}{d^2}\left(\frac{1}{2d}\right)\vec{\nabla}_r\left(d^2\right)$$

$$= \frac{1}{d^3}\left(\vec{r} + \vec{d_0}\right) = \frac{\vec{d}}{d^3} = \frac{\hat{d}}{d^2}$$

Giving:

$$\boxed{U_{Tid}(r) = -m G M_m \left(\frac{1}{d} + \frac{x}{d_0^2}\right)}$$

Difference in height between high, low tide (h). Sketch of greatly exaggerated tidal distortion.



$\bar{r} \approx R_E$ undistorted radius

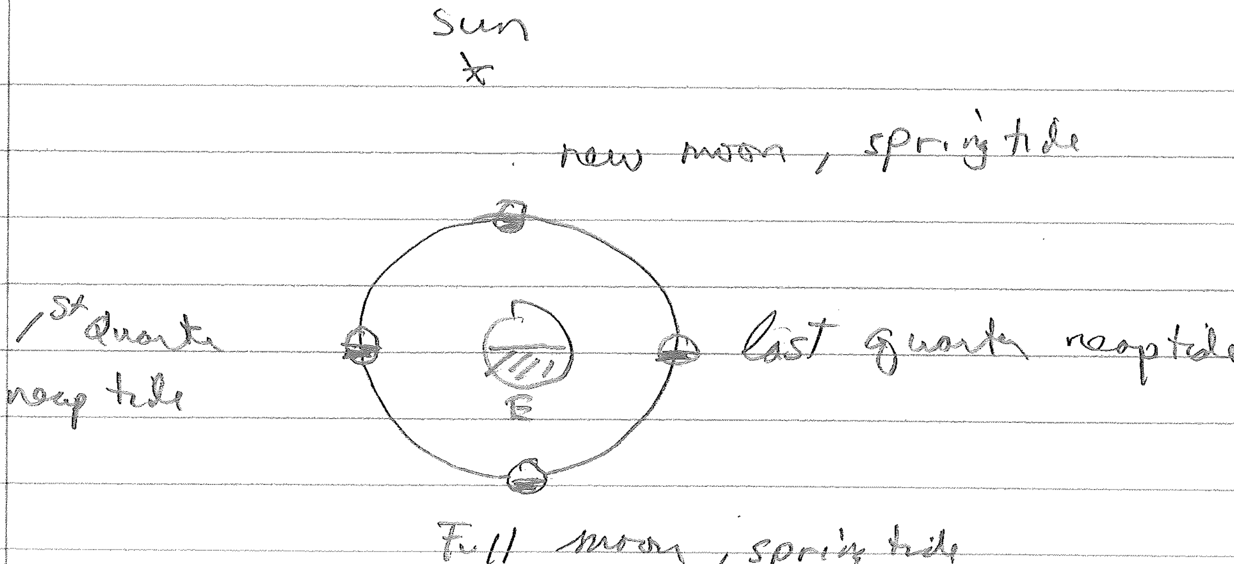
High tide point P at $\bar{r} + \frac{h}{2}$

low tide point Q at $\bar{r} - \frac{h}{2}$

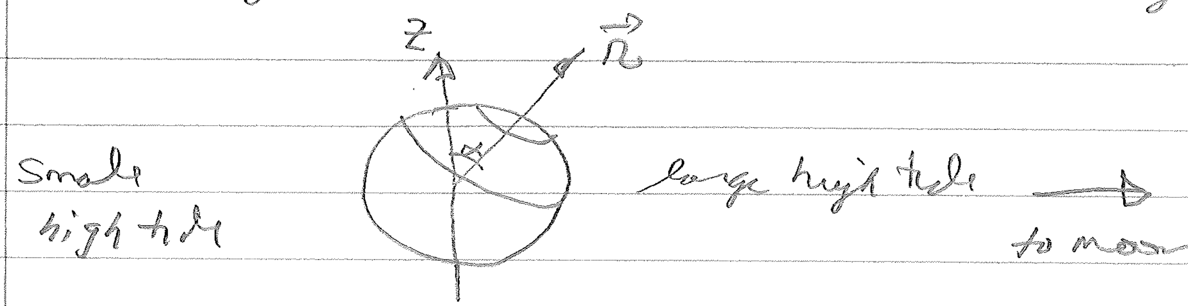
where r-bar = mean earth radius

Pages 5,6 have been redacted. see hw problem 9.6

[redacted]



Two high tide not the same due to tilt angle α



Other complications, e.g. continents

Friction slows earth's rotation by
$4.4\ 10^{-8}$ s/day

From conservation of angular momentum
E-moon system, moon's orbital L increasing
so E-m distance increases by 0.4 cm/month

Closest moon could ever have been is $\boxed{\text{Roche limit } 2.44 R_E}$
(moon destroyed by tidal forces)