Elastic Scattering Kinematics
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I did a Monte Carlo calculation of the scattering kinematics, generating events uniformly in the CM frame and making a Galilean transformation to the LAB frame.

1 Target mass m_2 twice projectile mass m_1)

We calculated the transform

$$\frac{d\Omega cm}{d\Omega_{LAB}}$$

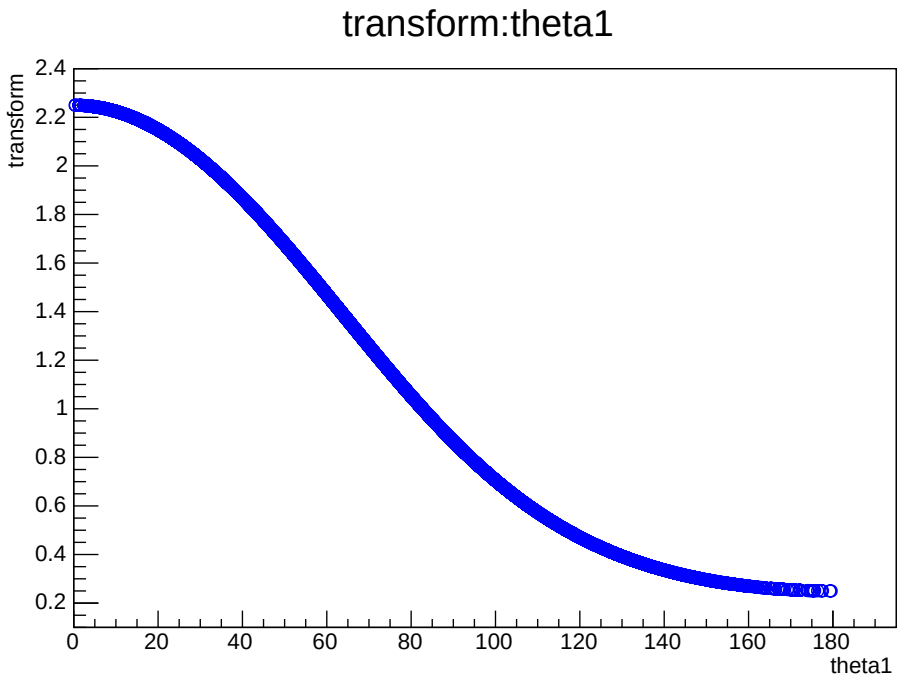

Figure 1: transform $m_2 = 2m_1$.

But it is really misleading (I would even say wrong) to plot $d\sigma/d\Omega_L$ versus θ. This is because the solid angle is

$$d\Omega = 2\pi \sin\theta d(\theta) = 2\pi d(\cos\theta)$$

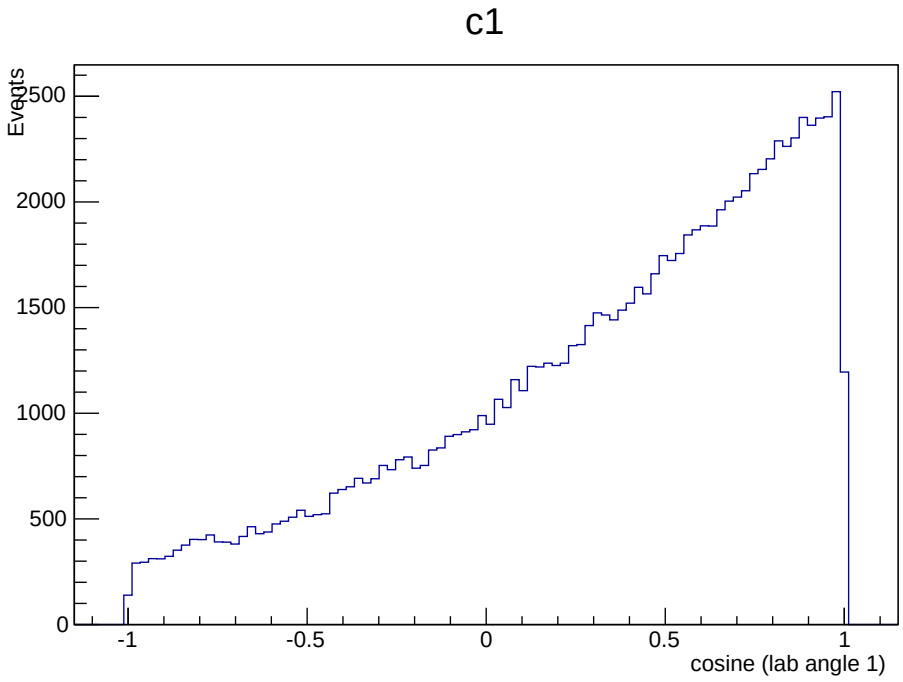

Figure 2: Here is $d\sigma/d\Omega_L$ versus $\cos\theta_L$, $m_2 = 2m_1$.

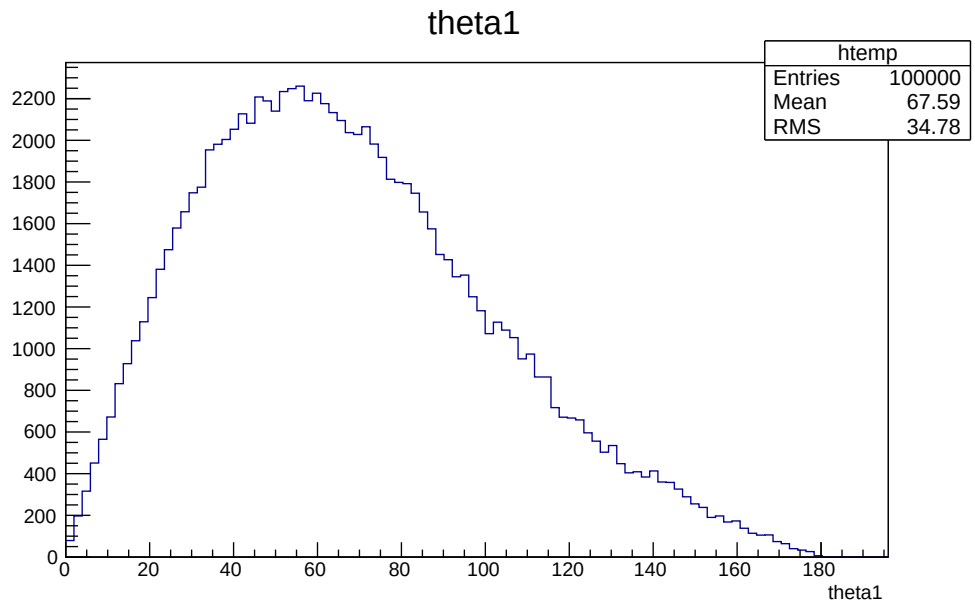

Figure 3: Here is $d\sigma/d\theta_L$ versus θ_L $m_2 = 2m_1$.

2 Equal Masses

For the special case of equal mass particles, it is easy to show that the angle between the scattered particles in the lab frome is 90 degrees.

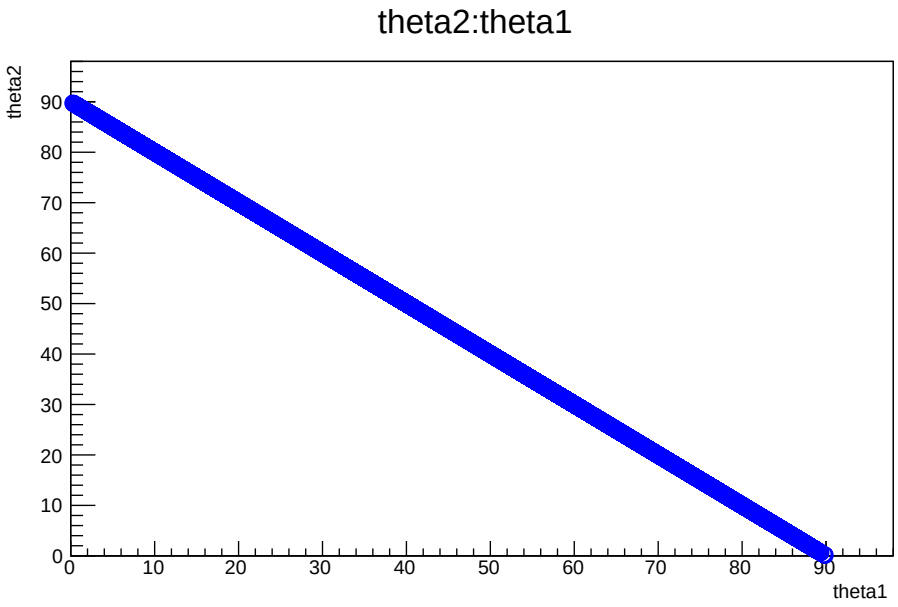

Figure 4: theta target versus theta projectile $m_1 = m_2$.

The cosine variable is easier to interpret because (with azimuthal symmetry) the solid angle is:

$$d\Omega = 2\pi \sin\theta d(\theta) = 2\pi d(\cos\theta)$$

We see that $\cos\theta_{CM}$ is indeed flat.

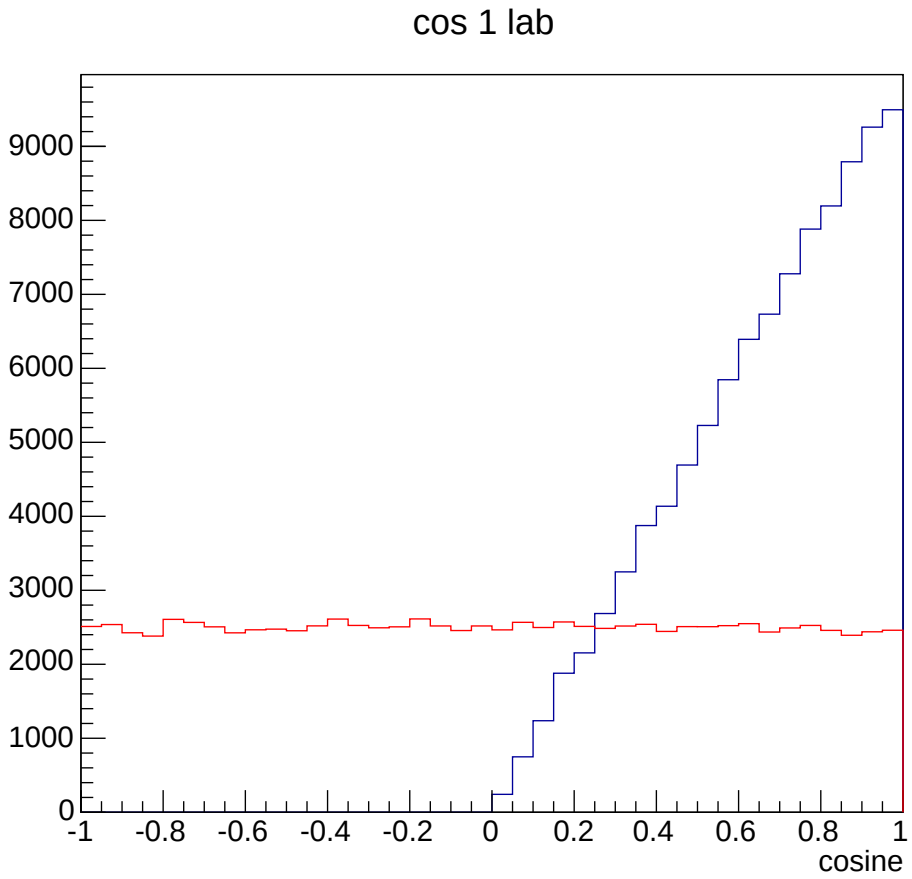

Figure 5: $\cos(\theta)$ for $m_1 = m_2$. Blue is lab, Red is CM frame

In terms of the angles, the cross section (cm,lab) looks like this:

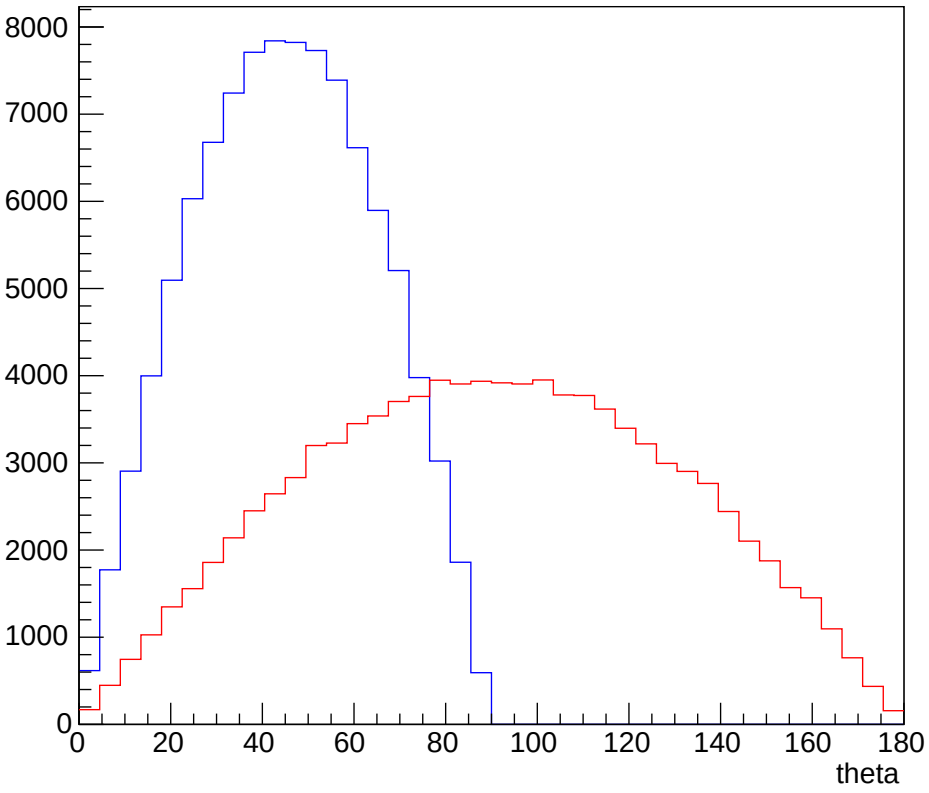

Figure 6: angle for $m_1 = m_2$. Blue is lab, Red is CM frame