

# The Lamb Shift

$$\Delta\nu = 1058 \text{ MHz} = 0.035 \text{ cm}^{-1}$$

# Deviations from Dirac theory observed by Houston and Williams

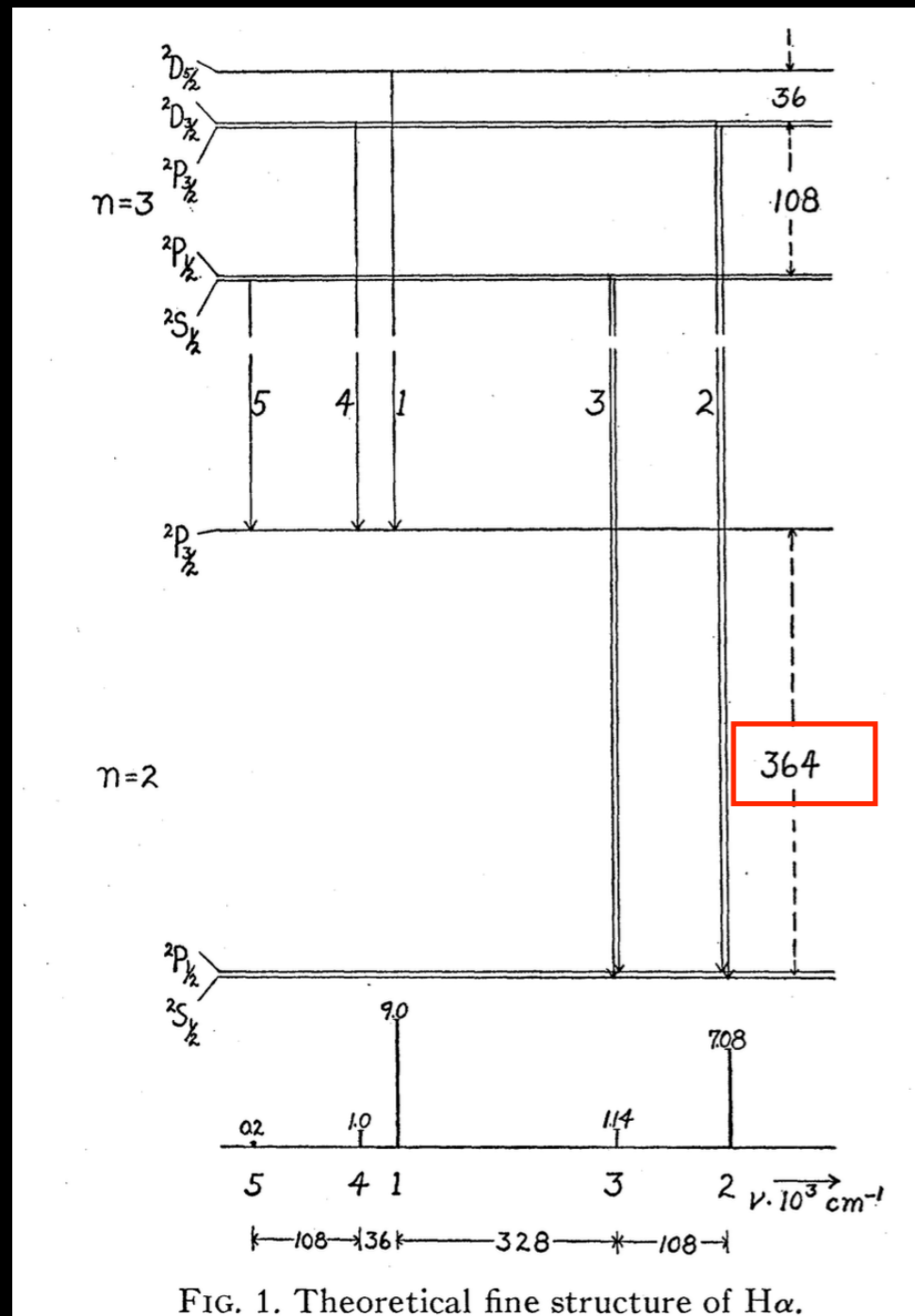


FIG. 1. Theoretical fine structure of H $\alpha$ .

Williams Phys.Rev. 54.558 (1938)

Dirac Theory  $2S_{1/2}, 2P_{1/2}$   
exact same energy

$2P_{3/2} \rightarrow 2P_{1/2}$

frequency  $\nu = 0.364 \text{ cm}^{-1}$

$\lambda = 2.74 \text{ cm}$  (micro-wave)

from Williams Phys.Rev. 54.558 (1938)

$n=3$  to  $n=2$  is  $H\alpha$

$D\alpha$  deuterium  
line broadening is mostly  
doppler @100K

TABLE I. Recent measurements of the doublet interval in  $H\alpha$  and  $D\alpha$ .

INVESTIGATOR	YEAR	DOUBLET INTERVAL IN $\text{cm}^{-1}$	
		$H\alpha$	$D\alpha$
Houston and Hsieh <sup>1</sup>	1934	0.312	
Williams and Gibbs <sup>2</sup>	1934	0.304	0.317
Kopfermann <sup>3</sup>	1934		0.323
Spedding, Shane and Grace <sup>4</sup>	1934	0.314	0.318
Williams and Gibbs (Unpublished)	Dec. 1934	0.314	0.318
Heyden <sup>5</sup>	1937		0.331

<sup>1</sup> Houston and Hsieh, Phys. Rev. **45**, 263 (1934).

<sup>2</sup> Williams and Gibbs, Phys. Rev. **45**, 475 (1934).

<sup>3</sup> Kopfermann, Naturwiss. **22**, 218 (1934).

<sup>4</sup> Spedding, Shane and Grace, Phys. Rev. **47**, 38 (1935).

<sup>5</sup> Heyden, Zeits. f. Physik **106**, 499 (1937).



FIG. 3. Interferometer fringes of  $D\alpha$  obtained with a 5 mm étalon spacing.

$$5\text{mm} = \Delta\nu \ 1 \ \text{cm}^{-1}$$

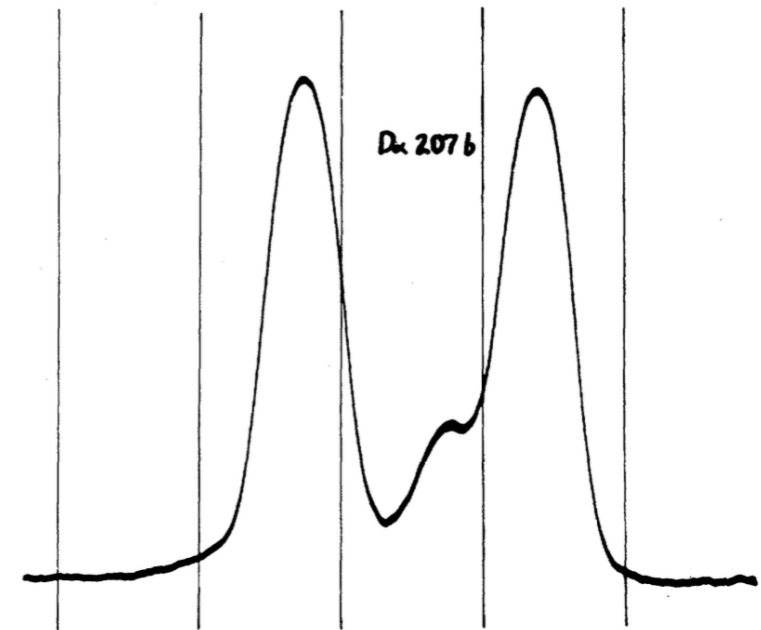


FIG. 4. Microphotometer tracing of part of the fringe pattern shown in Fig. 3.

Pasternak (1938) suggested that these results could be  $2S_{1/2}, 2P_{1/2}$  splitting of  $0.03 \text{ cm}^{-1}$ . However, most attributed discrepancies with Dirac to impurities in the source.

TABLE I.

	H $\alpha$	H $\beta$	H $\gamma$	H $\delta$	H $\epsilon$
$\Delta\nu_{\text{theor}}$	0.319	0.344	0.353	0.358	0.360
$\Delta\nu_{\text{obs}}$	0.307	0.330	0.339	0.345	0.351
$\Delta\nu_{\text{corr}}$	0.308	0.330	0.339	0.343	0.345

3rd row: if  $2S$  level displacement was  $0.030 \text{ cm}^{-1}$

accounting for H $\alpha$  line being from two transitions: ratio of intensities =  
 $\text{Intensity}(3D_{3/2}-2P_{1/2})/\text{Intensity}(3P_{3/2}-2S_{1/2})=2.4$

## Fine Structure of the Hydrogen Atom by a Microwave Method\* \*\*

WILLIS E. LAMB, JR. AND ROBERT C. RETHERFORD

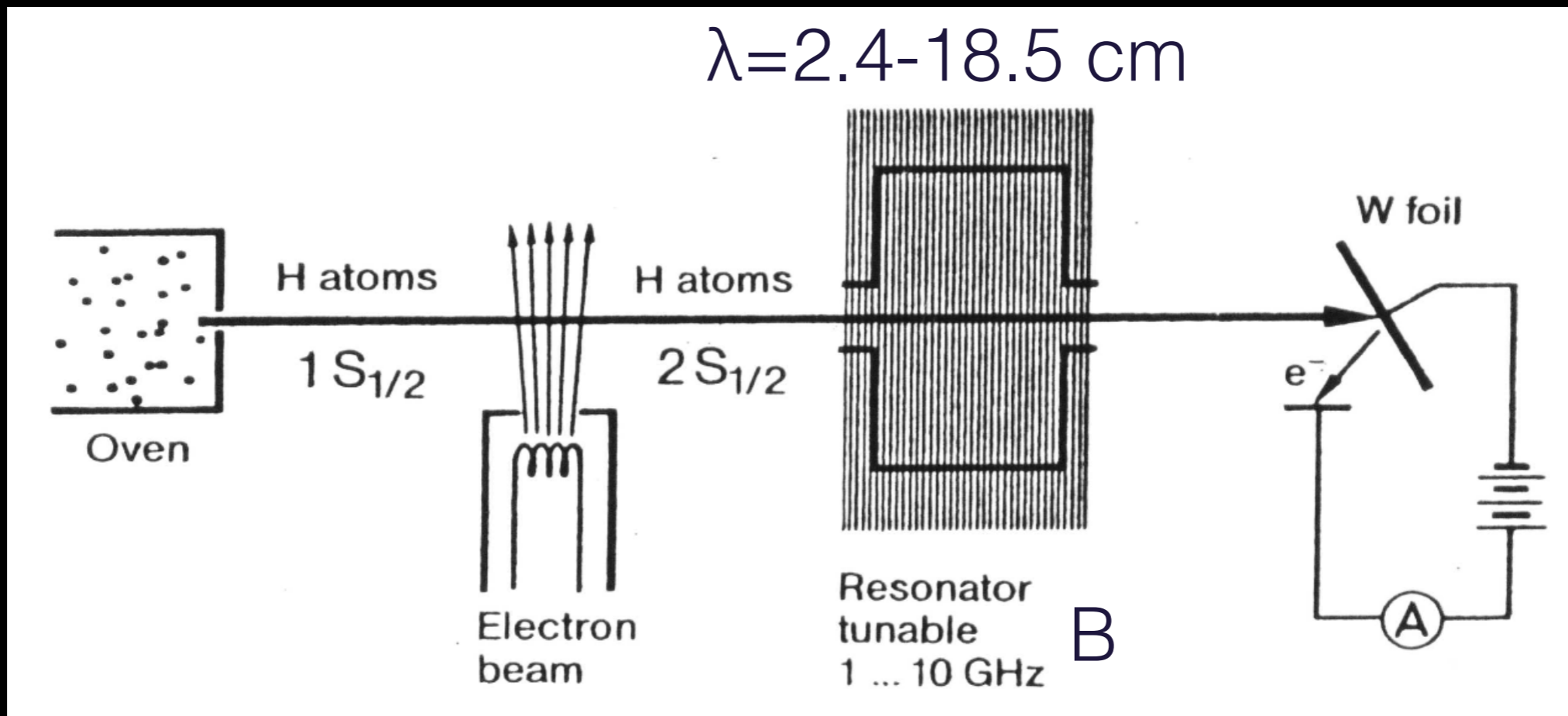
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(Received June 18, 1947)

THE spectrum of the simplest atom, hydrogen, has a fine structure<sup>1</sup> which according to the Dirac wave equation for an electron moving in a Coulomb field is due to the combined effects of relativistic variation of mass with velocity and spin-orbit coupling. It has been considered one of the great triumphs of Dirac's theory that it gave the "right" fine structure of the energy levels. However, the experimental attempts to obtain a really detailed confirmation through a study of the Balmer lines have been frustrated by the large Doppler effect of the lines in comparison to the small splitting of the lower or  $n = 2$  states. The various spectroscopic workers have alternated between finding confirmation<sup>2</sup> of the theory and discrepancies<sup>3</sup> of as much as eight percent. More accurate information would clearly provide a delicate test of the form of the correct relativistic wave equation, as well as information on the possibility of line shifts due to coupling of the atom with the radiation field and clues to the nature of any non-Coulombic interaction between the elementary particles: electron and proton.

radar technology  
developed for WWII

# schematic



Excited atoms (1/100 by e- beam) passed through a region containing both microwave radiation and an adjustable magnetic field, and then hit a metal target. Only excited atoms produce electrons upon hitting the W foil, producing current. The key to the experiment was that if the magnetic-field-induced energy difference between the two states was equal to the energy of the microwave photons, then the long-lived  $S$ -state would absorb a photon and turn into to the short-lived  $P$ -state. These atoms would drop back to their ground state before reaching the target, and the current in the detector would essentially vanish.

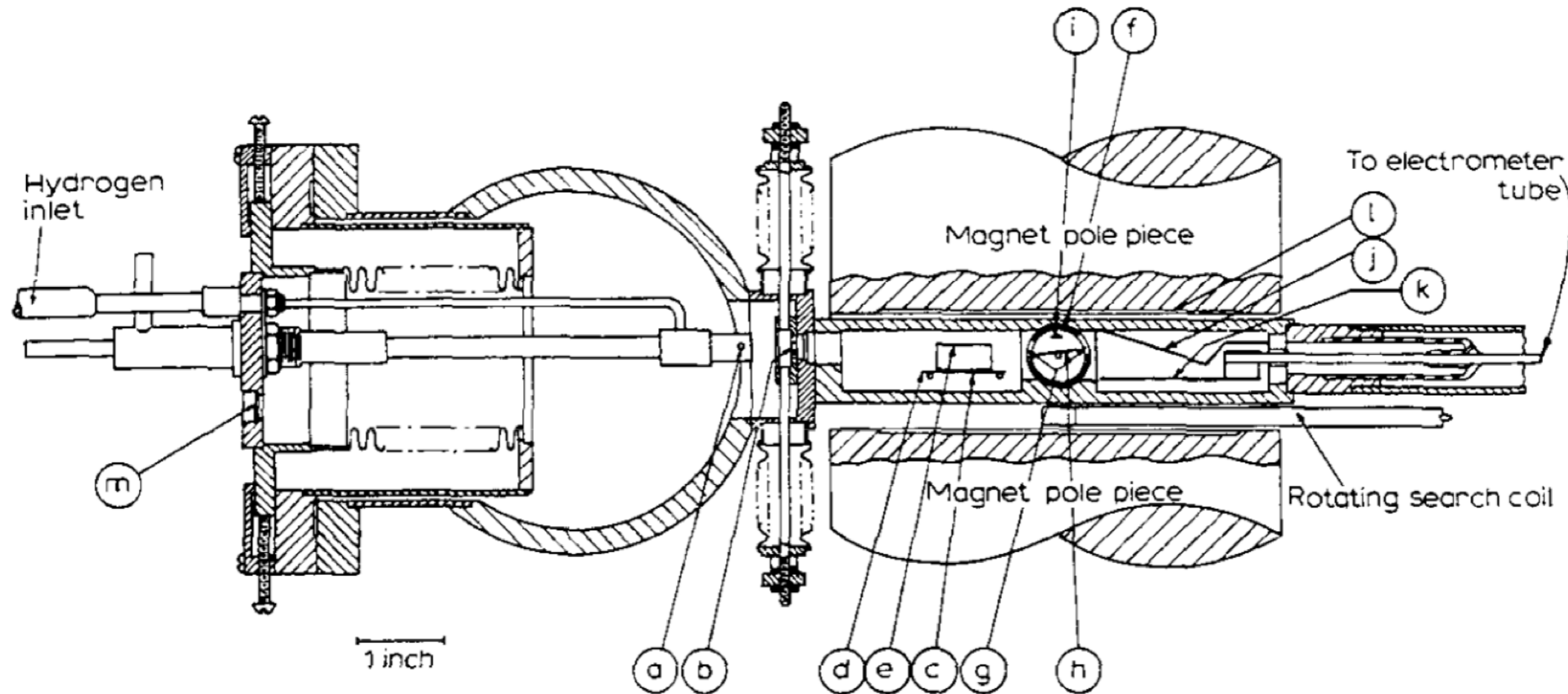


Fig. 3. Cross section of second apparatus: (a) tungsten oven of hydrogen dissociator, (b) movable slits, (c) electron bombarder cathode, (d) grid, (e) anode, (f) transmission line, (g) slots for passage of metastable atoms through interaction space, (h) plate attached to center conductor of r-f transmission line, (i) d.c. quenching electrode, (j) target for metastable atoms, (k) collector for electrons ejected from target, (l) pole face of magnet, (m) window for observation of tungsten oven temperature.

Lamb, 1947

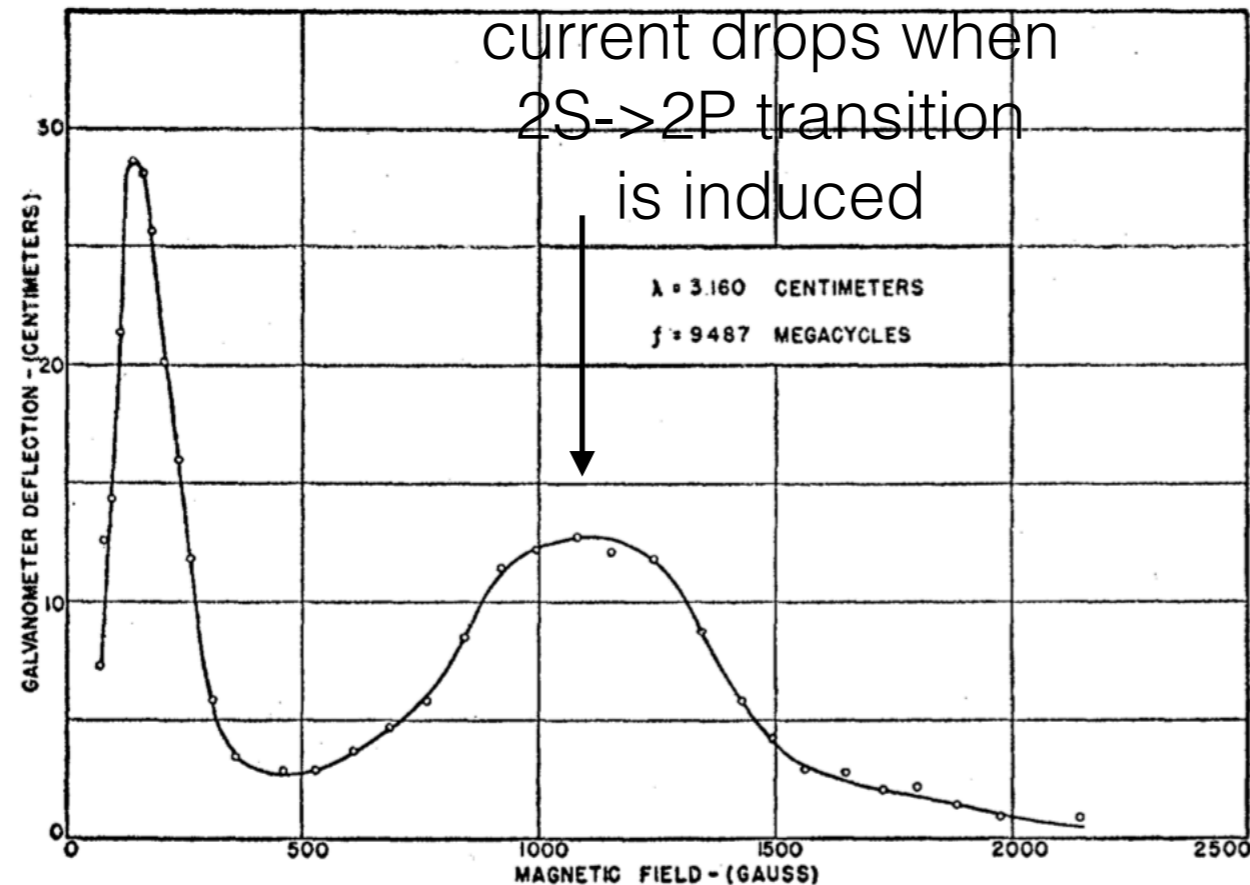
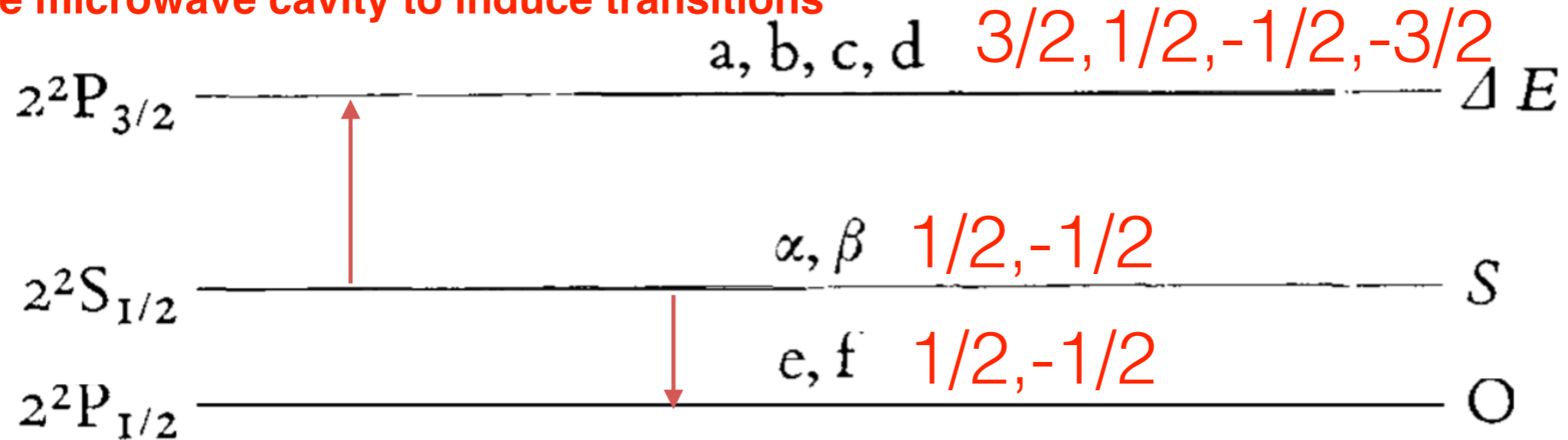


FIG. 1. A typical plot of galvanometer deflection due to interruption of the microwave radiation as a function of magnetic field. The magnetic field was calibrated with a flip coil and may be subject to some error which can be largely eliminated in a more refined apparatus. The width of the curves is probably due to the following causes: (1) the radiative line width of about 100 Mc/sec. of the  $^2P$  states, (2) hyperfine splitting of the  $^2S$  state which amounts to about 88 Mc/sec., (3) the use of an excessive intensity of radiation which gives increased absorption in the wings of the lines, and (4) inhomogeneity of the magnetic field. No transitions from the state  $2^2S_{\frac{1}{2}}(m = -\frac{1}{2})$  have been observed, but atoms in this state may be quenched by stray electric fields because of the more nearly exact degeneracy with the Zeeman pattern of the  $^2P$  states.



# Lamb Nobel 1955

tune microwave cavity to induce transitions



Electric dipole atomic transition rules for decay to ground state:

$\Delta l = 1, -$      $\Delta m_l = 1, 0, -$      $\Delta s =$     allowed  $\tau \sim ns$

$\Delta j = 1, 0, -1$     no  $j=0$  to  $j=0$      $2S_{1/2} \rightarrow 1S_{1/2}$      $\tau = 1/7 s$

$\alpha$  to  $a, b, c$      $\beta$  to  $b, c, d$      $\alpha, \beta$  to  $e$      $\alpha, \beta$  to  $f$

# Weak field Zeeman for Hydrogen varies linearly with B

290

1955 W.E.LAMB, JR.

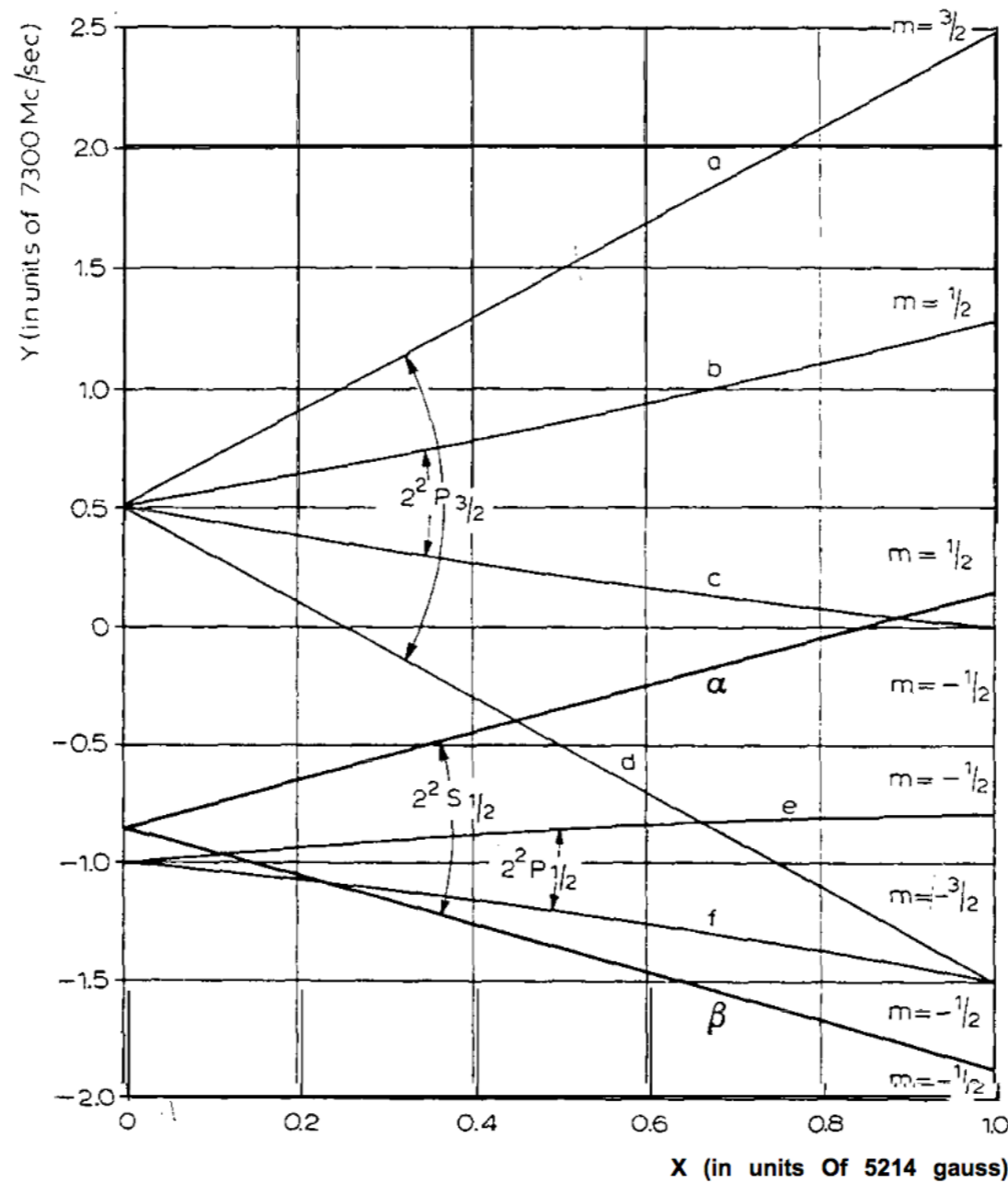


Fig. 4. Zeeman energy levels with the  $2^2 S_{1/2}$  pattern raised by 1000 Mc/sec.

Weak field Zeeman effect shift for Hydrogen in a magnetic field B is

$$\delta E = (\mu_{\text{Bohr}} B) \cdot m_j \cdot g_L(j, \ell)$$

Where  $m_j$  is the total angular momentum magnetic quantum number and the Lande g-factor is

$$g_L(j, \ell) = 1 + \frac{j(j+1) - \ell(\ell+1) + 3/4}{2j(j+1)}$$

Then we have,

- For  $2S_{1/2}$ ,  $g_L(1/2, 0) = 2$
- For  $2P_{3/2}$ ,  $g_L(3/2, 1) = 4/3$

Difference in transition energies from  $2P_{3/2, m_j} \rightarrow 2S_{1/2, 1/2}$  transition is

$$\begin{aligned} \frac{\Delta E}{\mu_{\text{Bohr}} B} &= \frac{4}{3} m_j - 1 \\ &= \begin{pmatrix} 1 \\ -\frac{1}{3} \\ -\frac{4}{3} \\ -3 \end{pmatrix} \end{aligned}$$

Lamb, 1947

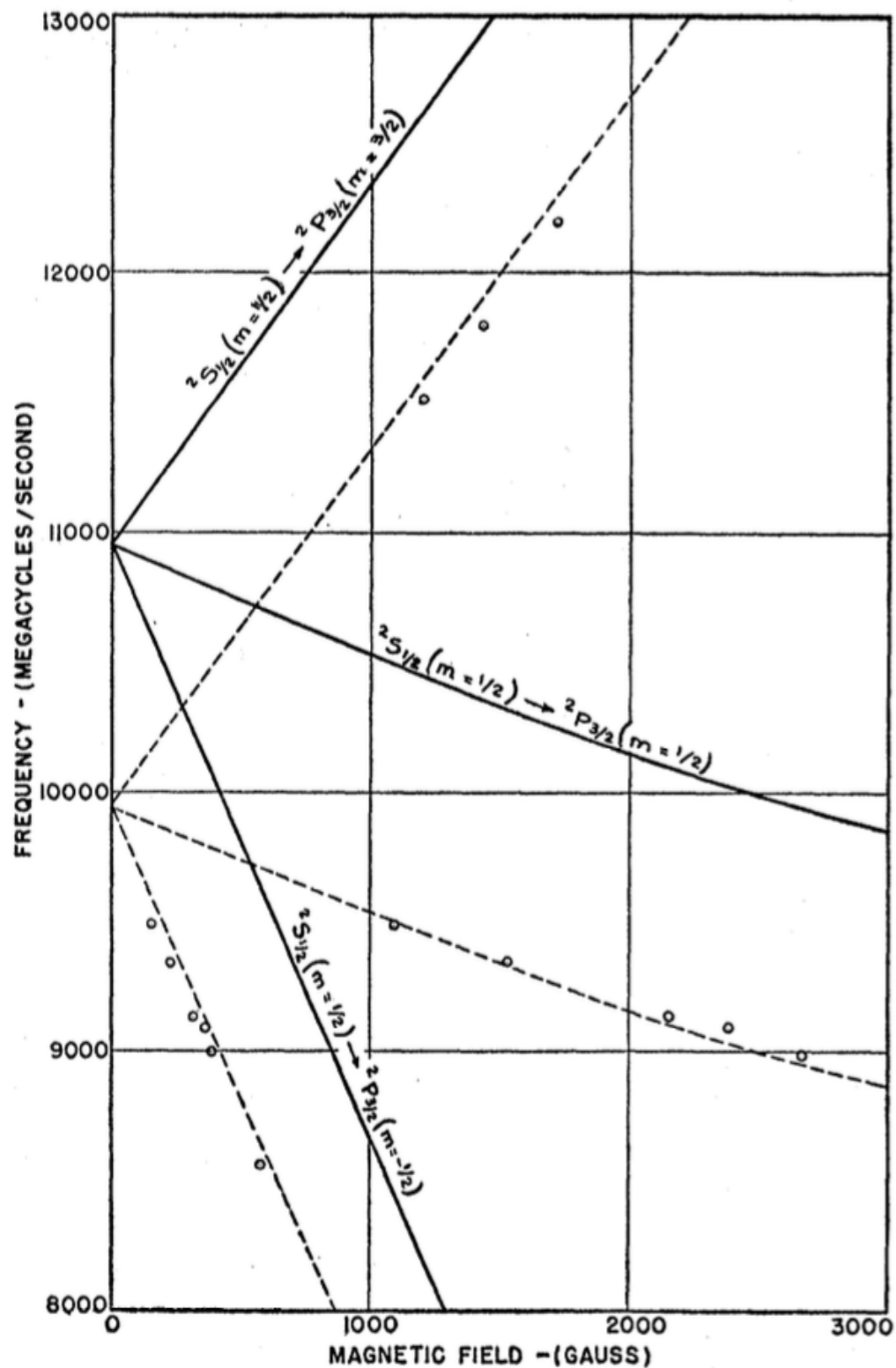
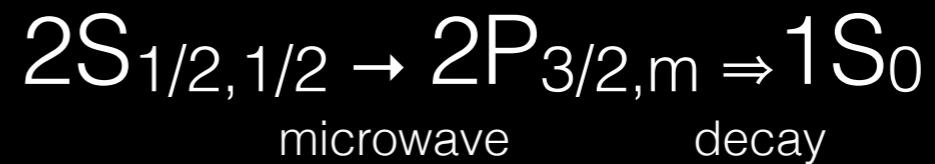
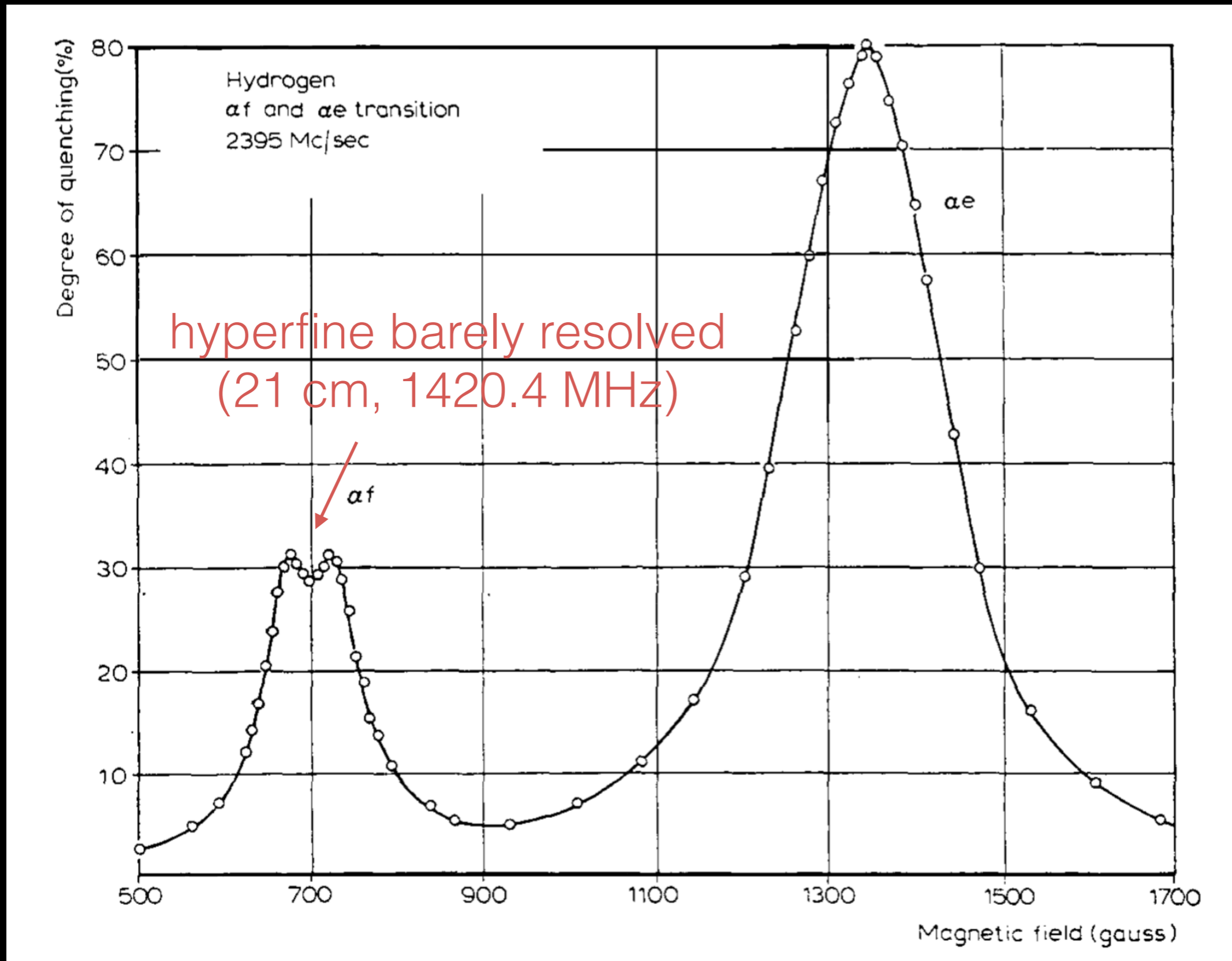


FIG. 2. Experimental values for resonance magnetic fields for various frequencies are shown by circles. The solid curves show three of the theoretically expected variations, and the broken curves are obtained by shifting these down by 1000 Mc/sec. This is done merely for the sake of comparison, and it is not implied that this would represent a "best fit." The plot covers only a small range of the frequency and magnetic field scale covered by our data, but a complete plot would not show up clearly on a small scale, and the shift indicated by the remainder of the data is quite compatible with a shift of 1000 Mc.

$$E(2S_{1/2}) > E(2P_{1/2})$$

# Lamb Nobel 1955



## The Electromagnetic Shift of Energy Levels

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(Received June 27, 1947)

**B**Y very beautiful experiments, Lamb and Retherford<sup>1</sup> have shown that the fine structure of the second quantum state of hydrogen does not agree with the prediction of the Dirac theory. The  $2s$  level, which according to Dirac's theory should coincide with the  $2p_{\frac{1}{2}}$  level, is actually higher than the latter by an amount of about  $0.033 \text{ cm}^{-1}$  or 1000 megacycles. This discrepancy had long been suspected from spectroscopic measurements.<sup>2,3</sup> However, so far no satisfactory theoretical explanation has been given. Kemble and Present, and Pasternack<sup>4</sup> have shown that the shift of the  $2s$  level cannot be

explained by a nuclear interaction of reasonable magnitude, and Uehling<sup>5</sup> has investigated the effect of the "polarization of the vacuum" in the Dirac hole theory, and has found that this effect also is much too small and has, in addition, the wrong sign.

Schwinger and Weisskopf, and Oppenheimer have suggested that a possible explanation might be the shift of energy levels by the interaction of the electron with the radiation field. This shift comes out infinite in all existing theories, and has therefore always been ignored. However, it is possible to identify the most strongly (linearly) divergent term in the level shift with an electromagnetic *mass* effect which must exist for a bound as well as for a free electron. This effect should

<sup>1</sup> Phys. Rev. **72**, 241 (1947).

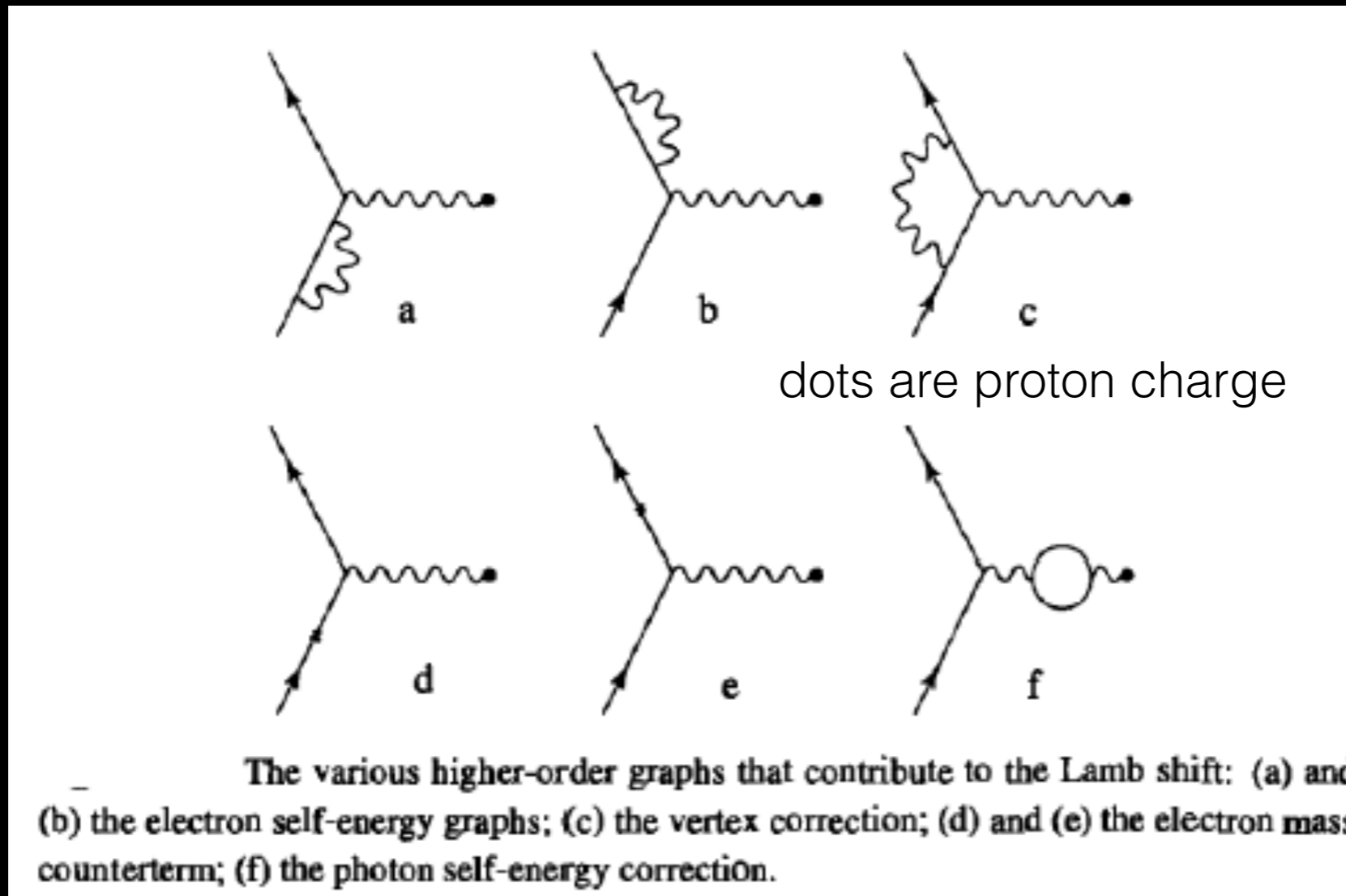
<sup>2</sup> W. V. Houston, Phys. Rev. **51**, 446 (1937).

<sup>3</sup> R. C. Williams, Phys. Rev. **54**, 558 (1938).

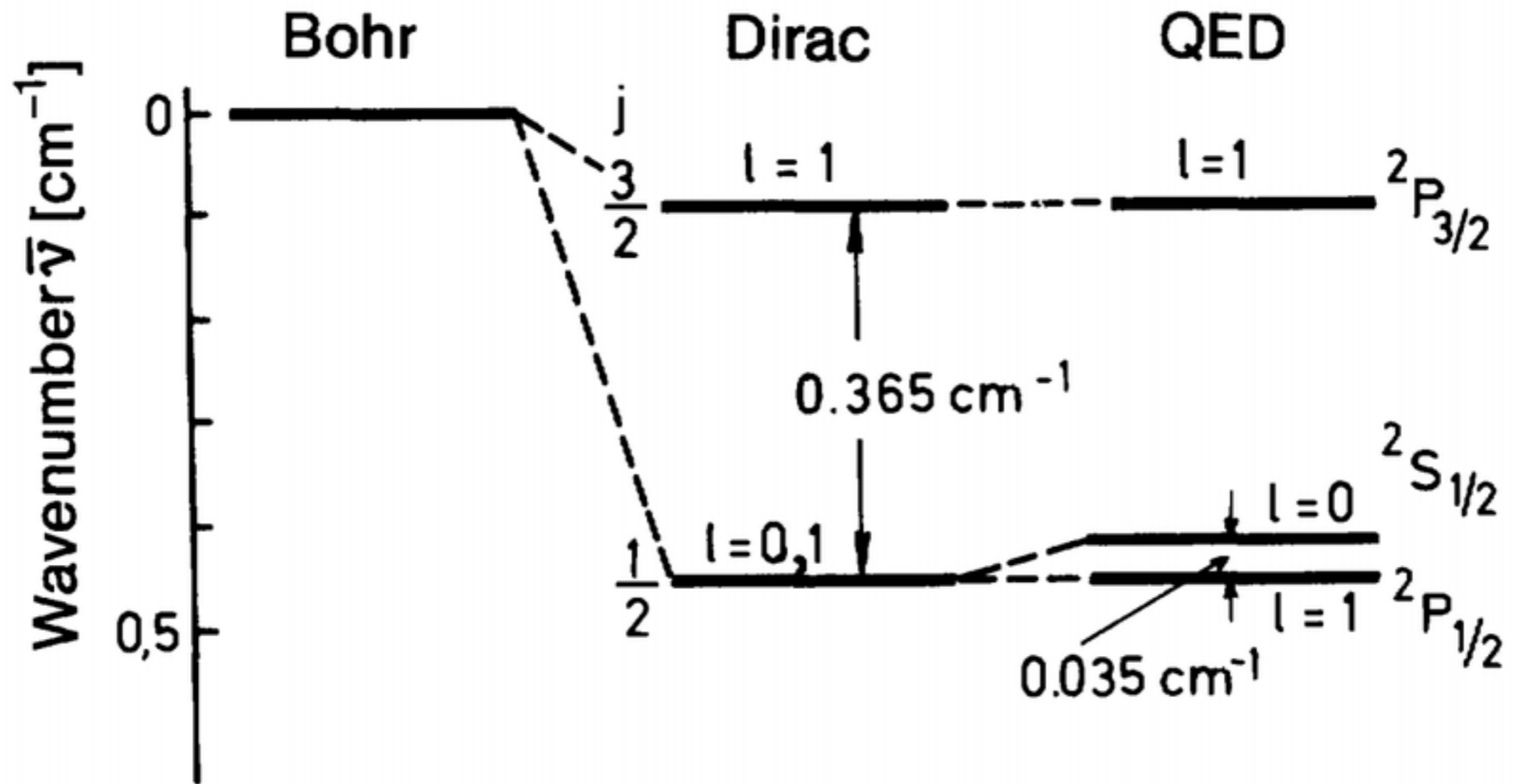
<sup>4</sup> E. C. Kemble and R. D. Present, Phys. Rev. **44**, 1031 (1932); S. Pasternack, Phys. Rev. **54**, 1113 (1938).

<sup>5</sup> E. A. Uehling, Phys. Rev. **48**, 55 (1935).

# Lamb shift in Quantum Electrodynamics is due to vacuum fluctuations



$\propto$  to probability for  $e^-$  to be at  $r=0$  (S states)





## Last word on Lamb shift:


-Lamb shift is precision test of QED

PRL 68 1120 (1992)

$$\Delta E_L^{\text{exp}} / h (2S-2P) = (1057.845 \pm 0.009) \text{ MHz}$$

$$\Delta E_L^{\text{th}} / h (2S-2P) = (1057.874 \pm 0.018) \text{ MHz}$$

Theory limited by uncertainty in  $\langle r \rangle$  proton (exp).

Exp. limited by (irreducible) 2p natural 

line width,

theory depends on exp. proton radius