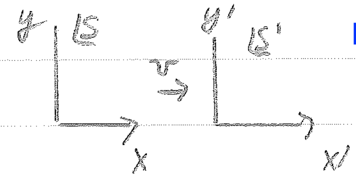
Phys 330 Spring 2020

Hw #1 Solutions

(11) relative sync. of clocks

a) 

$$\Delta t' = \gamma(\Delta t - \beta \Delta x)$$

v is boost to frame S'

if $\Delta t = 0, \Delta x = 0$ $\quad \Delta t' = 0$

$$\boxed{\text{if } \Delta t = 0, \Delta x \neq 0 \quad \Delta t' = -\gamma\beta \Delta x}$$

b) Since $\Delta x = 0$ even if $\Delta y \neq 0, \Delta z \neq 0$, $\Delta t' = 0$

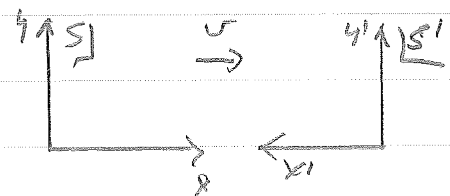
c) $t' = -\gamma\beta x$ $\quad t' < 0$ for $x > 0$

$\quad t' > 0$ for $x < 0$

d) $t = \gamma(t' + \beta x')\big|_{t'=0} = \gamma\beta x'$ $\quad t > 0 \quad x' > 0$

inverse transform

$\quad t < 0 \quad x' < 0$

e) suppose 

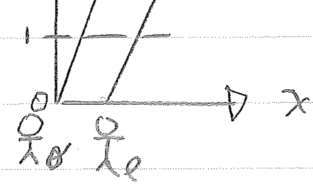
Viewed in S' frame S moves in $+x'$ direction.

So L.T. in each direction is the same.

$$\Delta t' = -\gamma\beta \Delta x$$

$$\Delta t = -\gamma\beta \Delta x' \qquad \Delta x' \text{ flips sign}$$

f) To measure clocks to be out of synchronization requires measuring at least two clocks and therefore two observers separated in space in frame S.



$$t'_0 = \gamma(t - \beta x)\big|_{x=0} = \gamma t$$

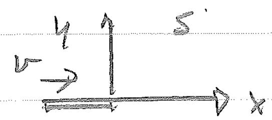
$$t'_l = \gamma(t - \beta x)\big|_{x=l} = \gamma t - \gamma\beta l$$

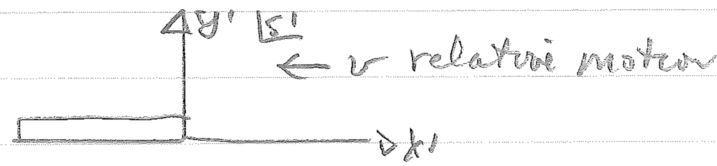
$t'_l - t'_0 = -\gamma\beta l$ moving clocks separated in space do not measure the same time.

"Two rocket observers separated in space along the direction of relative motion by comparing times on two moving clocks will observe that moving clocks are not synchronized."

(13) moving stick in frame S

Lorentz Contraction





Stick at rest in frame S'. Take $t = t' = 0$, $x = x' = 0$ when front of stick passes both origins. let event B be when back of stick passes $x = 0$.

$$x_B' = -l_0$$
$$t_B' = l_0/v \qquad \text{proper length}/v$$

L.T.
$$t_B = \gamma(t_B' + v x_B') = \gamma\left(\frac{l_0}{v} + v(-l_0)\right)$$
$$x_B = \gamma(x_B' + v t_B') = \gamma\left(-l_0 + v\left(\frac{l_0}{v}\right)\right) = 0$$

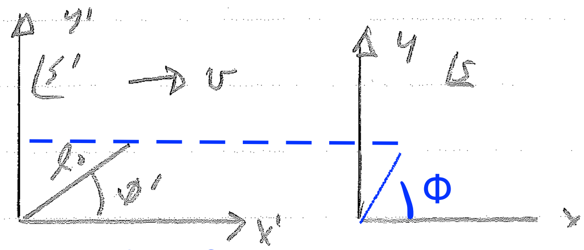
$$t_B = \gamma \frac{l_0}{v}(1 - v^2) = \frac{1}{\gamma}\left(\frac{l_0}{v}\right) \qquad \frac{\text{contracted length}}{v}$$

length in frame S is $\frac{l_0}{\gamma}$

LT of angles

HW #1-3

(19)



meter stick at rest in frame S'

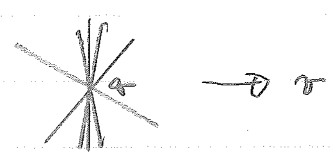
$\Delta x = \frac{1}{\gamma} \ell_0 \cos\phi'$ $\quad\ell = \sqrt{(\Delta x)^2 + (\Delta y)^2}$

$\Delta y = \ell_0 \sin\phi'$ $\quad = \frac{\ell_0}{\gamma}\sqrt{\cos^2\phi' + \gamma^2 \sin^2\phi'}$

didn't need l

$\tan\phi = \frac{\Delta y}{\Delta x} = \gamma \tan\phi'$

field lines of moving charge are closer together along directions ⊥ relative motion

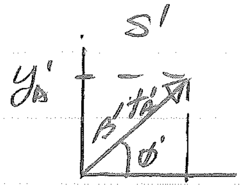


Force ⊥ to $\vec{v}$ > then when charge is at rest
Force ∥ " < "

This explains "relativistic rise" in charged particle energy loss observed for fast particles moving through a material.

Transformation of velocity direction

(21)



$$x_A' = \cos\phi' \, (\beta' t_A')$$
$$y_A' = \sin\phi' \, (\beta' t_A')$$

point after traveling time tA' in frame S'

$$x_A = \gamma\,(x_A' + \beta t_A') = \gamma\, t_A' \,(\beta' \cos\phi' + \beta)$$
$$y_A = y_A' = \sin\phi' \, \beta' \, t_A'$$

$$\tan\phi = \frac{y_A}{x_A} = \frac{\beta' \sin\phi'}{\gamma\,(\beta'\cos\phi' + \beta)}$$

in limit $\beta' = 1$, $\quad \tan\phi = \dfrac{1}{\gamma} \dfrac{\sin\phi'}{(\cos\phi' + \beta)}$

Compare to (19) for tilted meter stick

$$\tan\phi_s = \gamma \tan\phi_s'$$

$$\boxed{\text{in limit } \beta \to 1, \quad \phi_s \to 90^\circ \quad \text{whereas} \quad \phi \to 0}$$

field lines, relativistic rise

head light effect observed in neutral pion decay to 2 photons

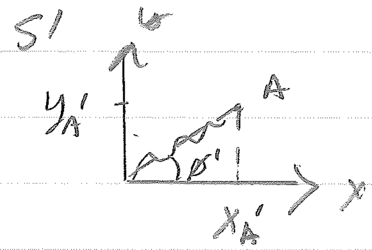
Headlight Effect

(22) Choose origins S, S' to coincide at $t=0$: @ $t=t'=0$, $x=x'=0$.

event A light has gone from origin to

$$x_A' = t_A' \cos\phi' \qquad y_A' = t_A' \sin\phi'$$

as always, take speed of light $= 1$



distance light travels is t_A'

Corresponding point in frame S -

$$x_A = \gamma(x_A' + \beta t_A') = \gamma t_A' (\cos\phi' + \beta)$$

$$t_A = \gamma(t_A' + \beta x_A') = \gamma t_A' (1 + \beta\cos\phi')$$

$$\cos\phi = \frac{x_A}{t_A} = \frac{\cos\phi' + \beta}{1+\beta\cos\phi'}$$

light travels at same speed in both frames

$$\sin\phi = \frac{y_A}{t_A} = \frac{\sin\phi'}{\gamma(1+\beta\cos\phi')}$$

$$\tan\phi = \frac{\sin\phi'}{\gamma(1+\beta\cos\phi')}\left(\frac{1+\beta\cos\phi'}{\cos\phi'+\beta}\right) = \frac{1}{\gamma}\frac{\sin\phi'}{(\cos\phi'+\beta)}$$

note: agrees w/ (21) in limit $\beta' \to 1$.