Physics 330 Spring 2020

HW #1

1. Relative synchronization of clocks

(a) Show that if two events occur simultaneously *and at the same place* in the laboratory frame they will occur simultaneously in all rocket frames. Show that if two events occur simultaneously in the laboratory frame but *not at the same position* on the x axis of the laboratory frame, they will *not* be simultaneous as observed in any moving rocket frame. The fact that observers in relative motion do not always agree whether two events are simultaneous is called the *relativity of simultaneity*.

(b) Two events occur simultaneously and at the same x coordinate in the laboratory frame, but are separated by the y and z coordinates Δy and Δz. Show that these two events are also simultaneous in the rocket frame.

(c) Use the Lorentz transformation equation to show that at $t = 0$ in the laboratory frame the clocks along the positive x axis in the rocket frame appear to be set behind those in the laboratory frame, with clocks farther from the origin set farther behind; and that clocks along the negative x axis in the rocket frame appear to be set ahead of those in the laboratory frame, with clocks farther from the origin set farther ahead, according to the equation

(46) $$t' = -x \sinh \theta_r = -x \beta_r/(1 - \beta_r^2)^{1/2}$$

(d) Use the inverse Lorentz transformation equation to show that at $t' = 0$ in the rocket frame the clocks along the positive x axis in the laboratory frame appear to be set ahead of those in the rocket frame, with clocks farther from the origin set farther ahead; and that clocks along the negative x axis in the laboratory frame appear to be set behind those in the rocket frame, with clocks farther from the origin set farther behind, according to the equation

(47) $$t = +x' \sinh \theta_r = +x' \beta_r/(1 - \beta_r^2)^{1/2}$$

The fact that neither of two observers in relative motion agrees that the reference event and the reading of zero time on all clocks of the *other* frame occur simultaneously is called the *relative synchronization* of clocks.

(e) The difference in sign between the equations in parts c and d seems to imply an asymmetry between frames that might be used to tell them apart—which would violate the principle of relativity. Show that if an observer in *either* frame chooses his positive x axis to lie in the direction of motion of the other frame, then physical measurements on the synchronization of clocks will give results in the two frames which are indistinguishable. In other words, the two frames themselves are indistinguishable using this method. The difference in sign between the above equations is due to an arbitrary—and asymmetric—choice of a *common* direction for both positive x axes.

(f) The foregoing results are sometimes summarized by stating that a "rocket observer sees the laboratory clocks to be out of synchronism with one another." Explain what is wrong with this way of stating the matter. Show that a single rocket observer is not enough to make the required measurements. What is a sharp, clean, legalistically correct, and clear (even if considerably longer!) way to state the same result?

12. Euclidean analogies

(a) A straight rod lies in the xy plane of a Euclidean coordinate system. Draw a diagram showing the rod in the xy plane; label the projections of this rod on the x, y and x', y' axes. Spell out an explicit analogy between the x components of the length of this rod as measured in two rotated Euclidean coordinate systems and the different lengths of a moving rod observed in the laboratory frame and in the rocket frame in which the rod is at rest.

(b) Spell out an explicit analogy between time dilation and the y components of length of the rod of part a as observed in two rotated Euclidean coordinate systems. What are the Euclidean and Lorentz invariants?

(c) Spell out an explicit analogy between the relative synchronization of clocks and the case of two rotated Euclidean coordinate systems in which points on the positive x axis of *one* coordinate system have, say, a negative y coordinate in the *other* coordinate system (more negative for points farther from the common origin).

13. Lorentz contraction II

A meter stick lies along the x' axis and at rest in the rocket frame. Show that an observer in the laboratory frame will conclude that the meter stick has undergone Lorentz contraction if he measures how long it takes the meter stick to pass one of his clocks and multiplies this result by the relative velocity of the two frames.

14. Time dilation II

Two events occur at the same place but at different times in the rocket frame. Show that an observer in the laboratory frame will conclude that the time between

19.* Transformation of angles

A meter stick lies at rest in the rocket frame and makes an angle ϕ' with the x' axis. What angle ϕ does the same meter stick make with the x axis of the laboratory frame? What is the *length* of the meter stick as observed in the laboratory frame? Next *assume* that the directions of electric-field lines around a point charge transform in the same way as the directions of meter sticks that lie along these lines. Draw qualitatively the electric-field lines due to an isolated positive point charge at rest in the rocket frame as seen in (a) the rocket frame and (b) the laboratory frame. What conclusions follow concerning the forces exerted, in the laboratory frame, on stationary test charges that surround a charge moving in that frame?

20.* Transformation of y velocity

A particle moves with uniform speed $\beta^{y'} = \Delta y'/\Delta t'$ along the y' axis of the rocket frame. Transform the

components of y and t displacements using the Lorentz transformation equations. Show that the x component and the y component of the velocity of this particle in the laboratory frame are given by the expressions

$$\begin{aligned} \beta^x &= \tanh \theta_r \\ \beta^y &= \beta^{y'}/\cosh \theta_r \end{aligned} \tag{49}$$

21.** Transformation of velocity directions

A particle moves with a velocity β' in the $x'y'$ plane of the rocket frame in a direction that makes an angle ϕ' with the x' axis. Find the angle that the velocity vector of this particle makes with the x axis of the laboratory frame. (Hint: Transform displacements rather than velocities.) Why does this angle differ from that found in Ex. 19? Contrast the two results when the relative velocity between the rocket and laboratory frames is very great.

22.** The headlight effect

A flash of light is emitted at an angle ϕ' with respect to the x' axis of the rocket frame. Show that the angle ϕ that the direction of this flash makes with respect to the x axis of the laboratory frame is given by the equation

$$\cos \phi = \frac{\cos \phi' + \beta_r}{1 + \beta_r \cos \phi'} \tag{50}$$

Show that your answer to the previous exercise gives the same result when the velocity β' is given the value one. Now consider a particle at rest in the rocket frame that emits light uniformly in all directions. Consider the 50 percent of this light that goes into the *forward* hemisphere in the rocket frame. Also, assume that the rocket moves very fast relative to the laboratory. Show that in the laboratory frame this light is concentrated in a *narrow forward cone* whose axis lies in the direction of motion of the particle. This effect is called the *headlight effect*.