Modern Physics 330: HW # 12

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1. Mössbaur effect

   The ^{57}Fe excited nucleus decays to a photon. The difference between energy levels of the nuclei $\Delta = 14.4$ keV. The lifetime of the excited state $\tau = 10^{-7}s$. Use energy and momentum conservation to calculate the difference in photon energy from Δ, $\epsilon = \Delta - E_\gamma$. Compare ϵ to the natural line width and show that unless the ^{57}Fe cannot recoil because it is embedded in a crystal lattice, resonant absorption cannot occur.

2. Pound and Rebka

   The principle of the experiment was to compensate for the gravitional red shift by the doppler effect. The height of the absorber above the emitter was 22.6 m. Calculate the expected gravitational red shift. Calculate the relative velocity between emitter and absorber needed to cancel the gravitational red shift. Determine the ratio of the expected red shift to the natural line width $\Delta E_\gamma/\Gamma$. What relative speed between emitter and absorber is required to shift the photon far enough so as to get no resonance absorption? *Historical footnote: a small difference in temperature of 1 degree K induces lattice vibrations of the same size as the gravitational red shift. They corrected for their measured temperature differences. The need for this correction was discovered empircally. Can you think of how they might have discovered it?*

3. Calculate the branching ratio (fraction of all decays occuring in paticular channel to all decays) for $W^- \to e^- + \bar{\nu}_e$ keeping in mind that quarks come in three colors.

4. For a radiation dominated universe with $k = 0$ and $\Lambda = 0$, find a differential equation for the scale factor $a(t)$ and its time derivatives. Assume a form $a(t) = \alpha t^p$ (α is a constant) and determine the power p. Do the same for a matter dominated universe.

5. Suppose the dark matter of the universe was in the form of Jupiter-like objects ($M = 2 \times 10^{27}$ kg). What density of Jupiters is needed? How many Jupiters are needed per galaxy on the average? A typical galaxy has a mass of 10^{11} solar masses where the solar mass unit is $M_\odot = 2 \times 10^{30}$ kg.

6. It is observed that there are about seven times more protons than neutrons in the universe. (a) Use Maxwell-Boltzmann statistics to estimate the temperature (T) of the universe when the proton to neutron ratio was seven. (b) The ratio becomes frozen when deuterons form and neutrons no longer decay. Compare the peak photon energy at temperature T with the deuteron binding energy 2.2 MeV (Recall that the plack body spectrum peaks at $\mathrm{hf}_{\mathrm{peak}} = 2.82$ kT. The big bang model makes a predition for the amount of primordial deutrium in the universe.

7. Dark Matter

   Calculate the dependance of the dark matter density on distance r to the center of the $\rho_D(r)$. Assume a star at r in uniform circular motion. Take the mass to be the visible mass (stars) plus dark matter. Assume that the visible mass is located in the core, $r_c < r$ and that the core has constant density ρ_c.

   Consider the rotation curve for the galaxy in the figure. The visible matter (stars) in the galaxy core is $4 \times 10^{12} M_\odot$. Calculate the fraction total dark matter out to the maximum radius in Fgure1, relative to the core mass.

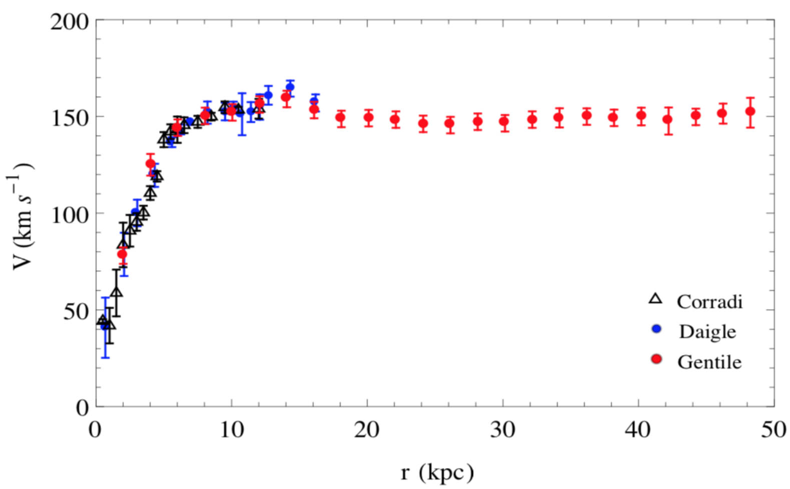

Figure 1: Rotation curve for galaxy NGC3198. From Karukes,Salucci1,and Gentile

8. Derive second Friedman equation given in lecture # 20.

9. Show that for photons the equation of state is $p = (1/3)\rho$. (Hint, see lecture # 15.)