Physics 330 Spring 2020

HW # 3 Solutions

#1

$\xrightarrow[p=200\text{ GeV}]{}$ $\cdot m$ $\qquad \tilde{P}_T = \begin{pmatrix} E \\ p \end{pmatrix}^{LAB} + \begin{pmatrix} m \\ 0 \end{pmatrix}^{LAB} = \begin{pmatrix} E+m \\ p \end{pmatrix}^{LAB}$

where $E = \sqrt{p^2+m^2}$ $\qquad \tilde{P}_T = \begin{pmatrix} E^* \\ 0 \end{pmatrix}^{CM}$

Use invariant interval

$$(E+m)^2 - p^2 = (E^*)^2$$

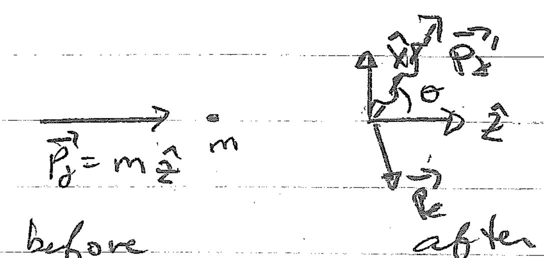
$$E^{*2} = (E+m)^2 - p^2 = \overset{p^2+m^2}{\overset{\parallel}{E^2}} + 2Em + m^2 - p^2$$

$$= 2Em + 2m^2 = 2m\left[\sqrt{p^2+m^2} + m\right]$$

now $m = 0.94$ GeV, $m \ll p$ so

$$E^* \cong \sqrt{2pm} = \sqrt{2(0.94)200} = 19.4 \text{ GeV}$$

#2



For $\theta = \frac{\pi}{2}$, conservation of momentum gives

$$\vec{P}_e = -P_\gamma'\,\hat{x} + P_\gamma\,\hat{z} \qquad P_e^2 = P_\gamma'^2 + P_\gamma^2$$

Conservation of energy gives

$$P_\gamma + m = P_\gamma' + E_e$$

now $P_\gamma = m$ so

$$P_e^2 = P_\gamma'^2 + m^2$$

$$2m = P_\gamma' + E_e$$

eliminate p_γ'

$$(p_\gamma')^2 = (2m - E_e)^2 = \overset{E_e^2 - m^2}{\overset{\shortparallel}{p_e^2}} - m^2 = E_e^2 - 2m^2$$

$$4m^2 - 4mE_e + E_e^2 = E_e^2 - 2m^2$$

$$6m^2 = 4mE_e$$

$$E_e = \frac{3}{2} m$$

#3

$$\xrightarrow{\quad\quad\quad} \quad \rightsquigarrow \quad\quad\quad \rightarrow \hat{x}$$

$E_e, \vec{P_e} = P_e \hat{x} \quad \vec{P_\gamma} = -P_\gamma \hat{x}$

$E_e = 10\,GeV = 10^{10}\,eV$

$m = \frac{1}{2} \times 10^6\,eV$

$P_\gamma = 10\,eV$ important ratio is -

$$\frac{m^2}{P_e P_\gamma} = \frac{\frac{1}{4}10^{12}(eV)^2}{10^{11}(eV)^2} = \frac{5}{2}$$

Boost to zero momentum (cm) frame

$$\begin{pmatrix} E_{cm} \\ 0 \end{pmatrix} = \gamma \begin{pmatrix} 1 & -v \\ -v & 1 \end{pmatrix} \begin{pmatrix} E_e + P_\gamma \\ P_e - P_\gamma \end{pmatrix}$$

$$-v(P_e - P_\gamma) + E_e + P_\gamma = 0 \quad\quad v = \frac{P_e - P_\gamma}{E_e + P_\gamma}$$

In cm frame before collision

$$P_\gamma^* = \gamma(1-v)(-P_\gamma)$$

after collision, backscattered photon momentum changes sign

$$P_\gamma^{*\prime} = \gamma(1-v) P_\gamma$$

Boost back to lab frame

$$P_\gamma' = \gamma(1-v)(1+v) P_\gamma = \left(\frac{1+v}{1-v}\right) P_\gamma$$

$$\frac{1+v}{1-v} = \left(\frac{E_e + P_\gamma + P_e - P_\gamma}{E_e + P_\gamma - P_e + P_\gamma}\right)$$

in denominator, Taylor expand $E_e = \sqrt{P_e^2 + m^2}$

$$= P_e\left(1 + \frac{1}{2}\frac{m^2}{P_e^2}\right) = P_e + \frac{1}{2}\frac{m^2}{P_e}$$

then $\left(\frac{1-v}{1+v}\right) = \frac{2P_e}{\frac{1}{2}\left(\frac{m^2}{P_e}\right) + 2P_\gamma}$

$$= \frac{P_e}{P_\gamma}\left(\frac{1}{\frac{1}{4}\frac{m^2}{P_e P_\gamma}+1}\right) = \frac{P_e}{P_\gamma}\left(\frac{1}{\frac{1}{4}\left(\frac{5}{2}\right)+1}\right) = \frac{P_e}{P_\gamma}\left(\frac{8}{13}\right)$$

then $\boxed{P_\gamma' = \frac{8}{13} P_e}$

<u>From $E, \vec{P}$ conservation in LAB frame</u>

$$E = E_e + P_\gamma = E_e' + P_\gamma'$$

$$P_e - P_\gamma = P_e' + P_\gamma'$$

$$(E_e - P_\gamma' + P_\gamma)^2 = E_e'^2 = P_e'^2 + m^2$$

$$(P_e - P_\gamma' - P_\gamma)^2 = P_e'^2$$

$$E_e \simeq P_e + \frac{1}{2}\frac{m^2}{P_e} = P_e + P_\gamma\left(\frac{5}{4}\right)$$

$$\left(P_e - P_\gamma' + \frac{9}{4}P_\gamma\right)^2 = P_e'^2 + m^2$$

$$(P_e - P_\gamma' - P_\gamma)^2 = P_e'^2$$

Subtract. neglect P_γ^2 compared to m^2

$$2(P_e - P_\gamma')\frac{9}{4}P_\gamma + 2(P_e - P_\gamma')P_\gamma \simeq m^2$$

$$(P_e - P_\gamma')\frac{13}{2}P_\gamma = m^2$$

$$P_\gamma' = P_e - \frac{2}{13}\frac{m^2}{P_\gamma} = P_e\left(1 - \frac{2}{13}\frac{m^2}{P_\gamma P_e}\right)$$

$$P_\gamma' = P_e\left(1 - \frac{2}{13}\left(\frac{5}{2}\right)\right) = \frac{8}{13}P_e$$

(#4) $\gamma + e^- \rightarrow e^- + e^- + e^+$

The minimum momentum in CM frame is $3m_e$.

Use invariance of interval

$$(E_\gamma + m)^2 - E_\gamma^2 = (3m)^2$$

$$2E_\gamma m + m^2 = 9m^2$$

$$E_\gamma = 4m$$

(#5) $\gamma + e^- \rightarrow e^-$

total $\tilde{P}_T = \begin{pmatrix} P_\gamma + m_e \\ P_\gamma \end{pmatrix} = \tilde{P}_T' = \begin{pmatrix} E_e \\ P_e \end{pmatrix}$ $\tilde{P}_T' \cdot \tilde{P}_T' = m_e^2$

after

invariant interval

$$(P_\gamma + m_e)^2 - P_\gamma^2 = m_e^2$$

$$2P_\gamma m_e = 0 \Rightarrow P_\gamma = 0$$

(#6) In π^0 rest frame, Photons have equal and opposite momentum $P_\gamma^* = \frac{m_\pi}{2}$

$\bar{P}_1^{cm} = \begin{pmatrix} P_\gamma^* \\ P_\gamma^* \end{pmatrix}$ $\bar{P}_2^{cm} = \begin{pmatrix} P_\gamma^* \\ -P_\gamma^* \end{pmatrix}$

Boost ($-\hat{x}$ direction) to lab frame.

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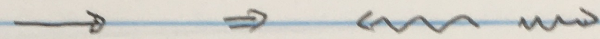
#6



$\vec{p}_\pi$ decay E_γ' E_γ

$$P_\pi = E_\gamma - E_\gamma'$$

if we wanted speed, $E = m\gamma = E_\gamma + E_\gamma'$

$p = m\gamma v = E_\gamma - E_\gamma'$

$$v = \frac{E_\gamma - E_\gamma'}{E_\gamma + E_\gamma'}$$

should have asked for speed.

The p^0 (energy) components are

$$p_{\gamma_1} = \gamma(1+\beta)\, p_\gamma^* \qquad p_{\gamma_2} = \gamma(1-\beta)\, p_\gamma^*$$

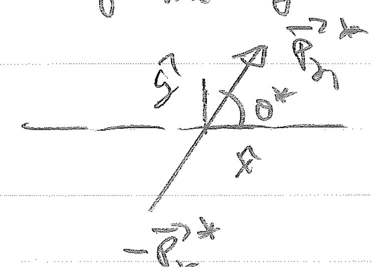
$$\frac{p_{\gamma 2}}{p_{\gamma_1}} = \frac{1-\beta}{1+\beta} \qquad \text{solve for } \beta$$

$$\beta = \frac{1 - p_{\gamma 2}/p_{\gamma 1}}{1 + p_{\gamma 2}/p_{\gamma 1}}$$

with corresponding γ,

$$p_{\pi^0} = \gamma\beta m_\pi$$

(#7) In rest frame of π^0



$$\vec{p}_{\gamma_1} = \frac{m_\pi}{2}\left(\cos\theta^* \hat{x} + \sin\theta^* \hat{y}\right)$$

Boost to lab in $-\hat{x}$ direction

$$\gamma = \frac{E_\pi}{m_\pi} = \frac{1\ \text{GeV}}{0.135\ \text{GeV}} \approx 7.4$$

$$\beta \cong 1$$

6

In the CM $\quad (c \equiv \cos\theta^* \quad s \equiv \sin\theta^*)$

$$\vec{P}_{\gamma_1} = \frac{m_\pi}{2}\left(\gamma(1+c)\,\hat{x} + s\,\hat{y}\right)$$

$$\vec{P}_{\gamma_2} = \frac{m_\pi}{2}\left(\gamma(1-c)\,\hat{x} - s\,\hat{y}\right)$$

$$\cos\alpha = \frac{\vec{P}_{\gamma_1}\cdot\vec{P}_{\gamma_2}}{|\vec{P}_{\gamma_1}|\,|\vec{P}_{\gamma_2}|} = \frac{\gamma^2(1-c^2) - s^2}{\left[\left(\gamma^2(1+c)^2+s^2\right)\left(\gamma^2(1-c)^2+s^2\right)\right]^{1/2}}$$

$\gamma=1$ cos = 0

$\gamma\rightarrow\infty$ cos = 1