Modern Physics 330: HW # 5

1. The group velocity of a wave packet is given by $v_g = d\omega/dk$, where the function $\omega(k)$ is called the dispersion relation. Show that the group velocity for a relativistic free particle $\hbar\omega = E = \sqrt{p^2 + m^2}$ is equal to the particle velocity.

2. Consider a particle confined to a box $-a/2 < x < a/2$ with wave function a t=0 given by the "tent" wave function

$$\psi(x,0) = A\left(\frac{a}{2} - |x|\right)$$

   for x inside the box and zero elsewhere.

   - Find the normalization constant A.
   - Find the uncertainty in x.
   - Is this an energy eigenstate?

3. Consider an energy eigenstate where the the spatial part $\phi_E(x)$ is real. Find $\langle p_x \rangle$.

4. Consider the wave function for a particle in a box $-a/2 < x < a/2$ given at $t = 0$ by

$$\psi(x,0) = A\cos\left(\frac{\pi x}{a}\right)$$

   - Find the normalization constant A.
   - Find the uncertainty in x.
   - Find the uncertainty in p_x and evaluate the uncertainty product $\Delta x \Delta p_x$.
   - Show that this an energy eigenstate and find the energy eigenvalue.

5. Following the proof of

$$\frac{d\langle x \rangle}{dt} = \frac{\langle p_x \rangle}{m}$$

   prove that

$$\frac{d\,\langle p_x \rangle}{dt} = \left\langle -\frac{\partial V}{\partial x} \right\rangle$$