Modern Physics 330: HW # 6

1. Consider a finite square-well for which the size of the potential is

$$Vo = \frac{1}{2m}\left(\frac{\epsilon\pi\hbar}{L}\right)^2$$

   where $\epsilon < 1$. Show that one and only one bound state exists. Find the approximate value of the energy of the bound state for $\epsilon \ll 1$.

2. An electron in a copper wire behaves like a free particle inside the wire. Estimate the distance that the electron wave function extends beyond the surface of the wire. The work function for copper is 4.7 eV.

3. Consider bound states of the "half-finite" square well potential, $V(x) = \infty$ for $x < 0$ $V(x) = -V_0$ for $0 < x < a$, and $V(x) = 0$ for $x > a$.

   - Sketch the ground state wave function.
   - What is the wave function for $x < a$? Find the minimum V_0 for a bound state to exist by requiring that the wave function "just turn over" at $x = a$. Is this what you expect from the symmetric square well solution?
   - The deuteron is very weakly bound. Assume the binding energy is 0 and calculate the minimum V_0. Take $a = 1.5$ fm. Here one should use the reduced mass of the two-body system, $\mu = m_p m_n/(m_p + m_n)$.
   - The deuteron binding energy is 2.2 MeV. Are the approximations made justified?

4. Consider a superposition harmonic oscillator state of the $n = 0$, $n = 1$ states defined at $t = 0$,

$$\phi = \frac{\sqrt{3}}{2}\phi_0 + \frac{1+i}{2\sqrt{2}}\phi_1$$

   - Prove that ϕ is properly normalized. (hint: use orthogonality)
   - Write $\Psi(x,t)$

- What is the probability of measuring the energy $\hbar\omega/2$ and $3\hbar\omega/2$? Does this probability vary with time?
- Find $\langle E \rangle$ and ΔE as functions of time.

5. The strong force binding quarks together can be modeled as the potential, $V(x) = b\,|x|$. Sketch the ground state and first excited state wave functions, and explain how you arrived at your sketch. Indicate the classical turning points. Estimate the ground state energy by using the uncertainty principle. Check that your answer has the dimensions of energy. (What are the dimensions of b?)

6. Consider the step function defined as $\Theta(x) = 0$ for $x < 0$ and $\Theta(x) = 1$ for $x > 0$. The derivative of $d\Theta/dx \equiv \delta(x)$, is a singular function (mathematically not a function at all, but a distribution) called the Dirac δ-function. Use integration by parts to prove that for any normalizable wave function $\psi(x)$,

$$\int_{-\infty}^{\infty} \delta(x)\psi(x)dx = \psi(0)$$