Modern Physics 330: HW # 7

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1. Show that the wave function
$$\psi = Ce^{-r/a_0}$$
where $a_0 = \hbar c/(mc^2\alpha)$ is a solution to the time independent Schrodinger equation for the Hydrogen atom. Determine the angular momentum quantum number ℓ and the energy eigenvalue. Show that the normalization constant $C = (\pi a_0^3)^{-\frac{1}{2}}$.

2. What is the approximate probability for the electron to be inside the proton in a Hydrogen atom? (Use $a_0 \gg 1$ fm.) What is the probability for the electron to be at a distance $> a_0$ from the proton?

3. In a more careful treatment, the the Schrodinger equation for Hydrogen has the electron mass replaced by the reduced mass $\mu = m_e m_p/(m_e + m_p)$. Deuterium is the stable isotope of Hydrogen with one proton and one neutron. Calculate the fractional difference of photon wavelengths due to photons emitted in the n = 2 → 1 transition coming from Deuterium compared to Hydrogen (Take $\mathrm{m_n} = \mathrm{m_p} = 10^3$ MeV and $\mathrm{m_e} = 0.5$ MeV).

4. Consider a model of an electron as a tiny uniform sphere of size 10^{18} meter corresponding to the experimental limit on possible electron structure. Suppose that the electron intrinsic angular momentum is due to a spinning motion of the sphere with an angular frequency ω. Calculate the value of ω. (Recall that the moment of inertia I of a sphere spinning about an axis passing through its center is $I = (2/5)mr^2$.) Calculate the speed of a point on the equator of the sphere.

5. In the Stem-Gerlach experiment, nonrelativistic silver atoms with a kinetic energy of E_k are sent through a nonuniform magnetic field that has a gradient (dB/dz) in the direction perpendicular to the initial trajectory of the silver atoms.

   - If the atoms pass a distance L through the magnetic field, use classical mechanics to describe the motoin and show that the beam is split by an amount
   $$x = \frac{\mu_B L^2}{4E_k}\left(\frac{dB}{dz}\right)$$

where μ_B the Bohr magneton.

- If the silver atoms come from an oven at a temperature of 1000 K, and they travel a distance of 0.05 m through the magnetic fi

  eld, calculate the magnetic

  eld gradient needed to make a splitting of 0.001 m. (Hint: The average kinetic energy of a particle escaping from an oven at temperature T is 2kT.)

6. **Wave Packets**

- Add two cosines with wave numbers $k - \Delta_k$ and $k + \Delta_k$ with $\Delta_k < k$ and show that the sum is

$$A = \frac{1}{2}\left[\cos\left(kx - \Delta_k x\right) + \cos\left(kx + \Delta_k x\right)\right] = \cos\left(kx\right)\cos\left(\Delta_k x\right)$$

  Define a parameter $a = k/\Delta_k$ Make a reasonable assumption for defining Δ_x and find the product $\Delta_x \Delta_k$ in terms of a.

- Plot the sum of two cosines for a = 2,5 and 20 over the range $-2aL < x < 2aL$.

- Instead of summing two cosines, sum n cosines, dividing Δ_k into n equal parts. Suitably normalize the sum. Plot over the range $-naL < x < naL$. Take $a = 20$ and $n = 5, 10$. Speculate what would happen as the sum becomes an integral.