Modern Physics 330: HW # 8

1. The fine structure splitting between the $2P_{3/2}$ and $2P_{1/2}$ multiplets is 4.52×10^{-5} eV. Calculate the fractional wavelength splitting $\Delta\lambda/\lambda$ in nanometers of the line from the $n = 2 \to n = 1$ transition.

2. What are the possible values for the total angular momentum quantum number j for an electron in a D ($\ell = 3$) state? What is the multiplicity for each of these j values, and show that the total number of states sums as it should.

3. Write the quantum numbers of the $n = 3$ states of hydrogen in terms of n, ℓ, m_ℓ, m_s and in terms n, ℓ, j, m_j. Write the energy degenerate muliplets $3L_j$ using spectroscopic notation (L=S,P,D..). Write the degeneracies of each multiplet, and show that they sum to the total you expect.

4. The Zeeman effect is the splitting of spectral lines when the atom is placed in an external magnetic field $\vec{B}_{ext}$. If the external field is large enough we can ignore the spin-orbit corrections and the shift in energy is (taking the magnetic field direction to be the z direction).

$$\Delta E = -\langle \mu_z \rangle B_{ext}$$

   The expectation value $\langle \mu_z \rangle$ can be written in terms of the orbital angular momentum magnetic quantum number m_ℓ and the spin magnetic quantum number m_s.

   (a) Show that

$$\langle \mu_z \rangle = \frac{e\hbar}{2m_e}\left[m_\ell + 2m_s\right]$$

   Be sure to say where the factor of 2 comes from.

   (b) Into how many states does the 3D state ofhydrogen divide in a strong external magnetic field?

   (c) Calculate the magnetic field strength needed to make an upward fractional energy shift ($\Delta E/E$) of 10^{-4} on the $3D_{5/2}$ state of hydrogen.

5. Suppose an electron is in the spinor state

$$\chi = C \begin{pmatrix} 1 \\ 1+i \end{pmatrix}_z$$

   (a) Find the normalization constant C.

   (b) Find $\langle \mu_z \rangle$.

6. For the ground state wave function of hydrogen, calculate the radial velocity of the electron as

$$v_r = \frac{1}{m}\sqrt{\langle \hat{p}_r^2 \rangle}$$

   The notation for the expectation value of any operater is defined as,

$$\langle \hat{O} \rangle = \int \psi^\star \hat{O} \psi d^3 r$$

   .

   There is one difficult integral but it is $\langle 1/r \rangle = 1/a_0$ where a_0 is the Bohr radius. Hint– For the radial momentum operator, choose the most convenient form from lecture 10.