Phy 330 Pandemic Spring 2020

HW #9 - Solutions

(#1)

$$\frac{4\langle \vec{S}_e \cdot \vec{S}_p \rangle}{\hbar^2} = j(j+1) - 2\left(\tfrac{1}{2}\right)\left(\tfrac{1}{2}+1\right)$$
$$= j(j+1) - 3/2$$

rule $|j_1 - j_2| \le j \le j_1 + j_2$
with $j_1 = j_2 = \frac{1}{2}$ gives $j = 0, 1$

$$\left.\begin{aligned} \Delta E_1^{hf} &= A/2 \\ \Delta E_0^{hf} &= -\tfrac{3}{2}A \end{aligned}\right\} \text{ difference } \Delta = 2A$$

$$\frac{\Delta}{|E_{10}|} = 2\left(\frac{2}{3}\right)2\alpha^2\left(\frac{m_e}{m_p}\right)g_p = \frac{8}{3}\left(\frac{1}{137}\right)^2\left(\frac{1}{1800}\right)5.59$$
$$= 4.41 \times 10^{-7}$$

$$\lambda = \frac{hc}{\Delta} = \frac{1240\ \text{eV}\cdot\text{nm}}{(13.6\ \text{eV})\,4.41\times 10^{-7}} = \underline{\underline{20.8\ \text{cm}}}$$

(2) Helium first excited state has 1 electron in 1S and 2nd in 2S or 2P. Spatial part is

$$\psi_{\pm}(1S,2S) = \frac{1}{\sqrt{2}}\left(\psi_{1s}(\vec{r}_1)\psi_{2s}(\vec{r}_2) \pm \psi_{1s}(\vec{r}_2)\psi_{2s}(\vec{r}_1)\right)$$

$$\psi_{\pm}(1S,2P) = \frac{1}{\sqrt{2}}\left(\psi_{1s}(\vec{r}_1)\psi_{2p}(\vec{r}_2) \pm \psi_{1s}(\vec{r}_2)\psi_{2p}(\vec{r}_1)\right)$$

+ symmetric, − antisymmetric

P state is triplet $m_l = 1, 0, -1$

Spin χ_0 antisymmetric singlet $m_s = 0$

χ_1 symmetric triplet $m_s = 1, 0, -1$

Wave function	multiplicity
$\psi_+(1S,2S)\,\chi_0$	1
$\psi_-(1S,2S)\,\chi_1$	3
$\psi_+(1S,2P)\,\chi_0$	3×1
$\psi_-(1S,2P)\,\chi_1$	3×3
	total 16

we must have $2(n^2)$ for each $n = 1, 2$:

$$2(n=1)^2 \times 2(n=2)^2 = 2\cdot2\cdot4 = 16$$

Sheet1

	A	B	C	D	E	F	G	H	I	J	K	L	M
1	0	1	2	3	4	5	6	7	8	9	10	mult	
2	4										1	5	5!/4!/1!
3	3	1								1		20	2x5!/3!/2!
4	3		1						1			20	
5	3			1				1				20	
6	3				1		1					20	
7	3					2						10	5!/3!/2!
8	2	2							1			30	5x(4!/2!)
9	2	1	1					1				20	
10	2	1		1			1					20	
11	2	1			1	1						20	
12	2		2				1					30	
13	2		1	1		1						20	
14	2		1		1	1						20	
15	2			2	1							30	
16	1	3						1				20	2x5!/3!/2!
17	1	2	1				1					20	
18	1	2		1		1						20	
19	1	2			2							20	5x(2x4!/2!)
20	1	1	1	1	1							120	5!
21	1	1		3								20	2x5!/3!/2!
22	1		2	2								30	
23	0	4					1					5	
24	0	3	1			1						20	
25	0	3		1	1							20	2x5!/3!/2!
26	0	2	2		1							30	
27	0	2	1	2								30	
28	0	1	3	1								20	2x5!/3!/2!
29	0		5									1	
30	42	32	22	16	9	7	5	3	2	1	1		
31													
32	920	740	515	480	300	120	95	60	50	20	5		

line 30 = indistinguishable Bose result, sum the columns

line 32 = distinguishable, multiply each row by multiplicity factor and then sum columns

	A	B	C	D	E	F	G	H	I	J	K
1	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
2	2	2							1		
3	2	1	1					1			
4	2	1		1			1				
5	2	1			1	1					
6	2		2				1				
7	2		1	1		1					
8	2		1		1	1					
9	2			2	1						
10	1	2	1				1				
11	1	2		1		1					
12	1	2			2						
13	1	1	1	1	1						
14	1		2	2							
15	0	2	2		1						
16	0	2	1	2							
17											
18	21	16	12	10	7	4	3	1	1		

Delete all lines with > 2 entries per box from Bose table

HW #9-4

② $\phi_1 = \sqrt{\frac{2}{L}} \cos\frac{\pi x}{L}$ $\quad \phi_2 = \sqrt{\frac{2}{L}} \sin\left(\frac{2\pi x}{L}\right)$

$$\Psi_\pm = \frac{1}{\sqrt{2}}\left(\phi_1(x_1)\phi_2(x_2) \pm \phi_1(x_2)\phi_2(x_1)\right)$$

$$|\Psi_\pm|^2 = \frac{1}{2}\Big[\phi_1^2(x_1)\phi_2^2(x_2) + \phi_1^2(x_2)\phi_2^2(x_1)$$

$$\pm 2\phi_1(x_1)\phi_2(x_1)\phi_1(x_2)\phi_2(x_2)\Big]$$

Since $|\phi_1|^2, |\phi_2|^2$ are normalized to 1 on the interval $-\frac{L}{2} < x < \frac{L}{2}$ and they are symmetric

$$P_\pm \equiv \int_0^{L/2}\int_0^{L/2} \Psi_\pm(x_1,x_2)\,dx_1\,dx_2 = \frac{1}{2}\left[\frac{1}{2}\cdot\frac{1}{2} + \frac{1}{2}\cdot\frac{1}{2} \pm 2J^2\right]$$

$$= \left[\frac{1}{4} \pm J^2\right]$$

where $J = \int_0^{L/2} \phi_1(x_1)\phi_2(x_1)\,dx_1$

$$= \frac{2}{L}\int_0^{L/2} \cos\frac{\pi x}{L}\sin\left(\frac{2\pi x}{L}\right)dx \qquad y = \frac{\pi x}{L}$$

$$= \frac{2}{\pi}\int_0^{\pi/2} \cos y \sin(2y)\,dy = \left(\frac{2}{\pi}\right)\left(\frac{2}{3}\right) = \frac{4}{3\pi}$$

then

$$P_{\pm} = \frac{1}{4} \pm \left(\frac{4}{3\pi}\right) = 0.25 \pm 0.18$$

$$= \begin{cases} 0.43 & \text{symmetric} \\ 0.07 & \text{antisymmetric} \end{cases}$$

For distinguishable particles,

$$P_D = \frac{1}{4}$$

distinguishable

Quantum statistics gives rise to <u>exchange force</u>

Bosons are pulled together

Fermions are pushed apart