

Modern Physics 330: HW # 9

1. Calculate the hyperfine transition line wavelength for the ground state of hydrogen. The hyperfine energy shift for the $n = 1$ state of hydrogen with total angular momentum quantum number j is

$$\Delta E_j^{hf} = A \frac{2\langle \vec{S}_e \cdot \vec{S}_p \rangle}{\hbar^2}$$

where

$$A = \frac{2}{3} mc^2 \alpha^4 \frac{m_e}{m_p} g_p$$

and where g_p is the g-factor for the proton. See lecture 13.

2. Write down the properly symmetrized wave functions (space) \times (spin) for the first excited state of helium. Use the notation:

$$\psi(1, S), \psi(2, S), \psi(2, P)$$

for the spatial parts of the electron wave functions and:

$$\chi_0, \chi_1$$

for the spin-0, spin-1 spin states. Give the multiplicity for each symmetrized wave function, and show that they sum to what you expect.

3. Consider a system of five particles sharing 10 units of energy. Calculate the mean occupation number $\bar{n}(n)$ for $n = 0, \dots, 10$. in the 3 cases: distinguishable, identical Bosons, identical Fermions. (Hint: use a spread sheet.)
4. Consider two identical particles in a one dimensional box of length L . Label the ground state as $\phi_1(x)$ and the first excited state $\phi_2(x)$. Write two possible wave functions, symmetrized and anti-symmetrized as $\Psi_{\pm}(x_1, x_2)$. Find the probability for both particles to be in the same half of the box in the two cases. What would you answer be if the particles were distinguishable? The difference for identical particles is referred to as an *exchange force*.