Lecture 10; Hatom Schrödungen in 3D: $\Psi(F,t) = e^{-iEt/t} \psi_{E}(F)$ 7 are energy eigenstate. $\vec{p} = \frac{\hbar}{i}\vec{p} = \hbar/i\left(\vec{e}_{1}\vec{e}_{2} + \vec{e}_{2}\vec{e}_{3} + \vec{e}_{2}\vec{e}_{3}\right)$ $\vec{p}^2 = [\vec{p}^2]^2 = \vec{p}_1^2 + \vec{p}_2^2 + \vec{p}_2^2 = -\vec{k}^2 \nabla^2.$ Laplacian 72 X B. V2 32+ 22+ 322 The porticle it of I = 2m I Hybrogen potential V(r) = - ke² = - tred is spherically symmetric. r à separation between proton and electron. Exploit spherical symmetry, J' in spherical coordinates (O = polar, p = azimuthel) S = Sinle $\nabla^2 = \frac{1}{7} \frac{\partial^2 r}{\partial r^2} + \frac{1}{12} \left(\frac{1}{500}\right) (sio) + \frac{1}{52} \frac{3}{082}$ $\frac{\hat{p}^2}{2m_e} = \frac{\hat{p}_e}{2m_e} + \frac{\hat{L}}{2m_e} \frac{\hat{r}^2}{2m_e}$ radial rotational

interpretation of operator, operating on furction. Fake J(r), ナディ(vf)=ナディ(F+レデ) =(デュ+ デラテ)+= (ビュデトジェ)ト du tiger v = (frat For) = (frat Sto) all equivalent Reguivalent operator expression. Pr= キ(ナジア) P2= - K2 + 22 r Tipie independent Schrodingin for My drozan n' IE= eithy $\left(\frac{P_{L}}{2m}+\frac{\Gamma^{2}}{2m}\right)$ $+\left(r,0,0\right)$ = E^{2} $+\left(r,0,0\right)$ V(r) E=0

10-2

22 indeid is expected operation (FYp)? D= Fxp = Fxq in spherical $\widehat{Z} = \frac{1}{2} \left(\widehat{\varphi} \, \widehat{\chi} \, \, \widehat{\chi$ in cartesian ! $\hat{\mathbf{L}} = \underline{\mathbf{T}} \hat{\mathbf{e}}_{\mathbf{x}} \left(- \partial \hat{\mathbf{m}} \phi \, \overline{\mathbf{3}} \phi - c_{\mathbf{0}\mathbf{1}} \phi \, \phi \, \mathbf{10} \, \overline{\mathbf{3}} \phi \right)$ + \$ \$ (corp 30 - miguto 37) + + & ? And (SESCHO) 22=-K2「+3=(55=)+ まう) Z, I have no lapenderce on r variable.

10-24

Separation of variable: Y(r,0,0) = R(-) Y(8,0) $\left(\frac{\hat{P}_{1}^{2}}{2m}+\frac{\hat{L}_{1}^{2}}{2m\gamma^{2}}-\frac{\pi hc}{F}\right)RY=ERY$ Y is called spherical harmonic and is solution of eigenvalue equation L'Y=t2CY c domensionleie We will see that C=R(R+1) R=0,1,2,5 Ground State Ground state Ray l=0. So -R2 (+ 2 (+ R)) - atc R = ER $2m \sqrt{r} \qquad A$ arggeste <math>U(r) = FR $-\frac{\pi^{2}}{2m} + \frac{1}{F}u'' - \alpha + tc \mu(r) = E U/r$ r^{2} $-\frac{k^2}{2m}u'' - \frac{xkc}{2}u = EU$ like 10 schrödingen except boundary Condition ((r=0) = 0.

Consider 1-700 Derricht $-\frac{t^2}{2m} \frac{y_0^{\mu}}{z} = E u_{eo} - E = |E| reg. for brund$ Spt $U_0'' = g^2 U_{00} \qquad \mathcal{B} = \left| \frac{2m|E|}{4^2} \right|$ $u(r) = e^{gr}$ Suggeste, let Ulrieg(r) e Br u'= (-82+9') e-85 u"= (-89'+9")e" - 8(-89+9')e" = (g"-28g'+82g)eor then et & cored giving - In (g"-28g'+gg) - theg = - 1E19 - 7 Carel 9"-299'+ (2mathc) 9 = 0 let g=Ar A=const g'=A note-U(r=0)=0 9"=0 (-28×+ 2m2C×=>)

10-5

g= km/El = mac 2m |E]= m2x22 => (E|= = = mc2x2 Ground statt everyte / E, = - zmc2k2 - 13.6ell) 6= the = Go Bohr radiu Normalization : R(+) = U(+) = Ae 4 = Ae 100 , Yoo = 195 $d\Omega = sin(theta)d(theta)d(phi)$ 1= Jr2d+ Ae Jan Yoo]2 1412 (Q6)32 JAN & - 1/20 = (7.43)1/2 C 1/20

10-6

10-7 V(r) = -2tic F rc/as 5 Ketch r/au Ę E VIV 15 $-\frac{dtc}{v_c} = -\frac{1}{2}mc^2 d^2$ rez 2 the = 2 Qu Radial probability density: P(+)dr= 417/4, 12r2dr=412-28/40 dr p(x)> Y/as

example Prob (r ≤ ao)

$$R_{ob} = \int_{0}^{a_{0}} r^{2}dr \int_{0}^{\pi} a_{0} \partial d\sigma \int_{0}^{2\pi} d\sigma |\vec{T}|^{2}$$

$$\vec{T}_{i} = e^{i\vec{F}_{i} + h_{i}} \left(\frac{1}{\pi\pi}\right)^{1/2} e^{-r/a_{0}}$$

$$\left|\vec{T}_{i}\right|^{2} = \frac{11}{\pi\pi}a_{0}^{3} e^{-2r/a_{0}}$$

$$R_{ob} = \frac{4\pi}{-\pi}a_{0}^{3} \int_{0}^{a_{0}} r^{2}dr e^{-2r/a_{0}} x = \frac{2\pi}{a_{0}}$$

$$= \frac{4\pi}{a_{0}^{3}} \left(\frac{a_{0}}{2}\right)^{3} \int_{0}^{2} x^{2}e^{-\chi}dx = \frac{1}{2}\left(0.647\right) = 0.32$$

$$= \frac{4}{a_{0}^{3}} \left(\frac{a_{0}}{2}\right)^{3} \int_{0}^{2} x^{2}e^{-\chi}dx$$

example

 $\langle r \rangle_{n=1} = \int \overline{\Psi} + \overline{\Psi} d^{3}r = \frac{4\pi}{\pi a_{0}^{3}} \int_{0}^{\infty} e^{-2r/a_{0}} r^{3} dr$ $=\frac{4}{q_{3}^{3}}\left(\frac{q_{0}}{2}\right)^{4}\int_{0}^{\infty}\chi^{3}e^{-\chi}d\chi = \frac{q_{0}}{4}3! = \frac{3}{2}G_{0}$

H atom wave function
Unem = Rn(r) Ven(O, Ø) = Un(r) Ven(O, Ø)
Three Queatum number
n = 1, 2, ... principal
l < n angular momentum
-l < m < l magnetic (integn stope)
degenonecy (2l+1) m valuer

$$\sum_{i} (2l+i) = n^{2}$$

Angular dependence Ven(O, Ø) aprears for
any rotationally mivariant potential
Rn(r) independent of l
special feature of \pm potential
 $V(r) = -\frac{wtc}{r}$

Modern Physics: Hydrogen Spectrum

Basic transitions given by Rydberg formula for transition from state $m \to n$

$$\frac{1}{\lambda} = \frac{|E_{1s}|}{hc} \left(\frac{1}{n^2} - \frac{1}{m^2}\right)$$

with $R_{\rm H} = \frac{|E_{1\rm s}|}{\rm hc} = \frac{13.6 eV}{1240 ev\cdot nm} = 0.0109737~\rm nm^{-1}$



Figure 1: H_{α} is first Balmer series line $3 \rightarrow 2$ with $\lambda_{\alpha} = 656.47$ nm



spectroscopic notation is: S(I=0) P(I=1) D(I=2) F(I=3)

Figure 2: Sketch of basic structure (no fine structure) of hydrogen. Orbital quantum number $\ell < n$ states are degenerate (same energy). The ground state n = 1 has only $\ell = 0$. Not shown are the magnetic quantum numbers for each ℓ , $-\ell \leq m \leq +\ell$ with multiplicity $2\ell + 1$. Total multiplicity for each n is $\sum_{1}^{n-1}(2\ell+1) = n^2$. Adding electron spin s = 1/2 with multiplicity 2s + 1 = 2 gives total multiplicity of each level $2n^2$.



Figure 3: Triangles mark $\langle r \rangle_{nl}$. Scale in units of Bohr radius ($a_0 = \frac{\hbar c}{mc^2 \alpha} = 0.05 \text{ nm}$)

Expectation value is given by

$$\langle r \rangle_{nl} = n^2 a_0 \left[1 + \frac{1}{2} \left(1 - \frac{l(l+1)}{n^2} \right) \right]$$

Note that the radial wave functions are "pushed out" due to orbital angular momentum.



Figure 7-3 The Coulomb potential V(r) and its eigenvalues E_n . For large values of *n* the eigenvalues become very closely spaced in energy since E_n approaches zero as *n* approaches infinity. Note that the intersection of V(r) and E_n , which defines the location of one end of the classically allowed region, moves out as *n* increases. Not shown in this figure is the continuum of eigenvalues at positive energies corresponding to unbound states.