Lec #13 : H atom relativistic corrections

$$\psi_{n\ell m_\ell m_s} = R_{n\ell}(r)\, Y_{\ell m_\ell}(\theta,\phi)\, \chi_{m_s}$$

electron spin quantum number $s = \frac{1}{2}$ which we don't write.

$$\hat{S}^2 \chi_{m_s} = \hbar^2 \tfrac{1}{2}\left(1+\tfrac{1}{2}\right)\chi_{m_s} = \hbar^2 \tfrac{3}{4}\chi_{m_s}$$
$$\hat{S}_z \chi_{m_s} = m_s \hbar \chi_{m_s}$$
} eigenvalue equations

aside

[formally, $\chi_+ = \begin{pmatrix}1\\0\end{pmatrix}$, $\chi_- = \begin{pmatrix}0\\1\end{pmatrix}$

$\hat{S}_z = \frac{\hbar}{2}\begin{pmatrix}1&0\\0&-1\end{pmatrix}$ and $\hat{S}^2 = \frac{3}{4}\hbar^2\begin{pmatrix}1&0\\0&1\end{pmatrix}$

with inner product $\chi^+ \equiv (C_+^*, C_-^*)$ so $\chi_m^* \chi_{m'} = \delta_{mn}$ orthonormal

and $\int \chi^+ \psi \, d^3\vec{r} = 1$]

relativistic corrections are small. electron speed v in H-atom $\langle v \rangle = \alpha c$ $\alpha = \frac{1}{137.036\ldots} = 0.007$

Expect shifts are $\Delta E \propto \alpha^2 E$
$\approx 5\times10^{-5}$

But measureable spectroscopically.

Predicted by relativistic Dirac equation.

Collectively, corrections are called "fine structure".