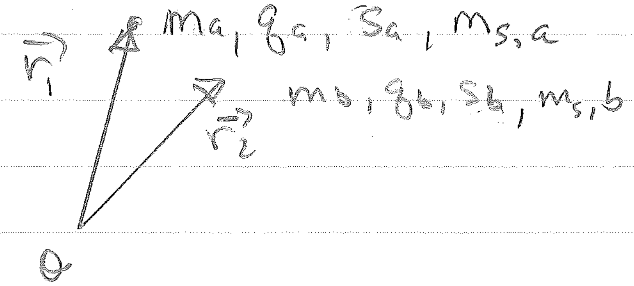
Phy 330 Spring 2020

Lec 14: Identical Particles

QM insists that multiparticle wave function <u>not</u> distinguish identical particles

Two Particle wave function:



<u>Distinguishable</u>: by mass or charge

$$\psi_{ab}(1,2) = \psi_a(\vec{r}_1)\chi_a^{(1)}\psi_b(\vec{r}_2)\chi_b^{(2)}$$

<u>Indistinguishable (identical)</u> same mass, charge

(probability density function)
P.D.F. may not allow a measurement to distinguish them. So P.D.F. must be symmetric under particle exchange — exchange symmetry

$$|\psi_{ab}(1,2)|^2 = |\psi_{ab}(2,1)|^2$$

leaves <u>two</u> possibilities:

$$\psi_{ab}(1,2) = \pm\,\psi_{ab}(2,1)$$

Spin-Statistics theorem of Quantum Field Theory and experimental fact:

$$\psi_{ab}(1,2) = +\psi_{ab}(2,1) \quad \text{integer-spin "Bosons"}$$

$$\psi_{ab}(1,2) = -\psi_{ab}(2,1) \quad \tfrac{1}{2}\text{ integer-spin "Fermions"}$$

Strikingly different statistical behavior:
Bose-Einstein, Fermi-Dirac

<u>Pauli Exclusion Principle</u>: No two electrons can occupy the same state

$$\psi_{ab} = \frac{1}{\sqrt{2}}\left[\psi_a(\vec{r}_1)\chi_a^{(1)}\, \psi_b(\vec{r}_2)\chi_b^{(2)} - \psi_a(\vec{r}_2)\chi_a^{(2)}\psi_b(\vec{r}_1)\chi_a^{(1)} \right]$$

explicitly zero if $a=b$.

How to build anti-symmetrized two electron wave function.

$$\psi_{TOT}(1,2) = \psi_{space}(\vec{r}_1,\vec{r}_2)\,\chi_{spin}$$

$$= \begin{cases} \psi^A_{space}\,\chi^S & A \equiv \text{anti-symmetric} \\ \psi^S_{space}\,\chi^A & S \equiv \text{symmetric} \end{cases}$$

General rule for combining angular momentum j_1 and j_2:

$$|j_1 - j_2| \le j \le j_1 + j_2$$

example, 2 spin $\frac{1}{2}$:

$$\left|\tfrac{1}{2} - \tfrac{1}{2}\right| \le j \le \tfrac{1}{2} + \tfrac{1}{2} \Rightarrow j = 0, 1$$

In group-theoretic notation, multiplets are denoted by degeneracy.

$$\underset{\sim}{2} \otimes \underset{\sim}{2} = \underset{\sim}{1} \oplus \underset{\sim}{3}$$

explicitly shows sum of states is the same

Spinor states $\chi(s, m_s)$

notation $\chi_1(\frac{1}{2}, \frac{1}{2}) \chi(\frac{1}{2}, -\frac{1}{2}) \equiv \uparrow\downarrow$

order indicates 1, 2

$\underset{\sim}{2} \otimes \underset{\sim}{2}$: $\uparrow\uparrow, \uparrow\downarrow, \downarrow\uparrow, \downarrow\downarrow$

$$\underset{\sim}{3}: \left.\begin{array}{l} \chi(1,1) = \uparrow\uparrow \\ \chi(1,0) = \frac{1}{\sqrt{2}}(\uparrow\downarrow + \downarrow\uparrow) \\ \chi(1,-1) = \downarrow\downarrow \end{array}\right\} \text{symmetric}$$

$\underset{\sim}{1}$: $\chi(0,0) = \frac{1}{\sqrt{2}}(\uparrow\downarrow - \downarrow\uparrow)$ antisymmetric

Group theoretic multiplets (irreducible representation) always have a definite symmetry.

<u>Irreducible</u> – under rotations, components of multiplets mix, but not between multiplets

example – 3 quarks

$$\underset{\sim}{2} \otimes \underset{\sim}{2} \otimes \underset{\sim}{2} = \left(\underset{\sim}{1} \oplus \underset{\sim}{3}\right)_{12} \otimes 2_3$$

$$= \underset{\sim}{2}_{\{1,2\}} \oplus \underset{\sim}{2}_{\{2,3\}} \oplus \underset{\sim}{4}_{(1,2,3}$$

antisymmetric — spin $\frac{1}{2}$ baryons p, n

symmetrized — $S = 3/2$ baryons Δ

<u>Helium</u>

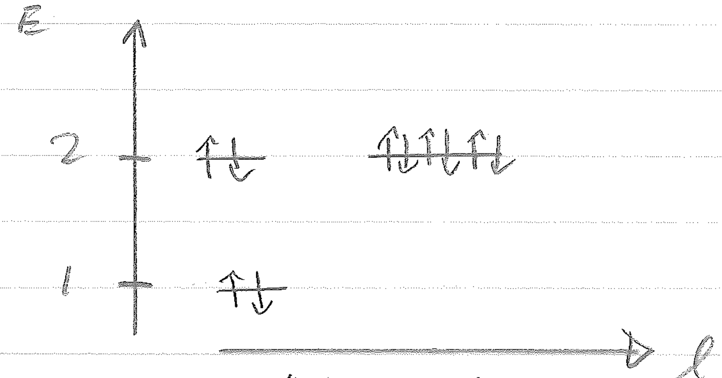
$$\psi^{\text{ground}} = \psi^S \chi^A(0,0)$$

$$\psi^S = \psi_{100}(\vec{r}_1)\psi_{100}(\vec{r}_2)$$

$$\chi_0 = \frac{1}{\sqrt{2}}(\uparrow\downarrow - \downarrow\uparrow)$$

Multi electron atoms - Shell model

electrons fill shells according to hydrogen principle quantum number n.



	0 (s)	1 (P)
$d_\ell = 2(2\ell+1)$	2	6

$d_n = 2n^2$

\# electrons

H $1s^1$

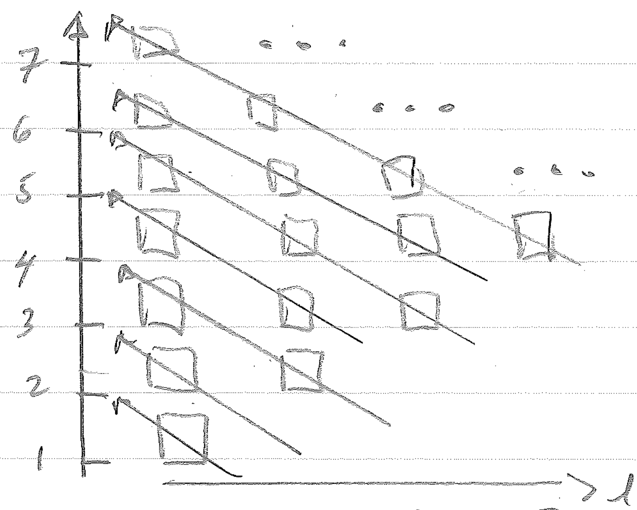
He $1s^2$ filled shell

Li $1s^2 2s^1$ . . .

Ne $1s^2 2s^2\ 2p^6$

<u>Order of filling:</u> see for example,

http://hyperphysics.phy-astr.gsu.edu/hbase/pertab/perfill.html#c2



	0	1	2	3
	s	p	d	f
# electron	2	6	10	20