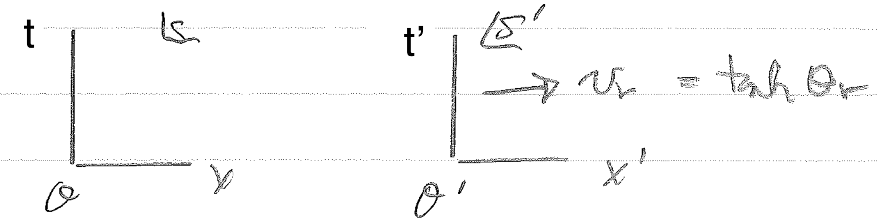
Phys. 330 Spring 2020

Lecture #2: Space-time

Space time diagram (Taylor & Wheeler, p. 48)



at $t = t' = 0$ origins coincide $x = x' = 0$ @ $t = t' = 0$

world line of origin O' in frame S

$$\begin{pmatrix} t' \\ 0 \end{pmatrix} = \begin{pmatrix} \cosh\theta_r & -\sinh\theta_r \\ -\sinh\theta_r & \cosh\theta_r \end{pmatrix} \begin{pmatrix} t \\ x \end{pmatrix}$$

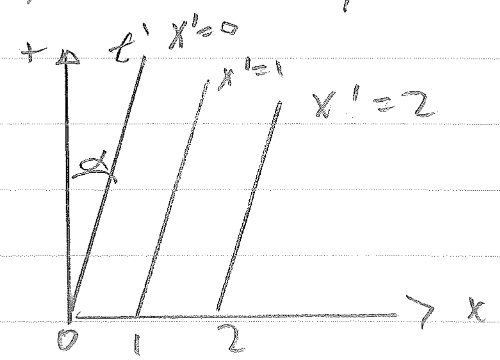
where we are measuring t in meters –

$$t = 1\,m = c\,t_{sec} \qquad t_{sec}(1\,m) = \frac{1\,m}{3\times10^8\,m/s} = \frac{10}{3}\,ns$$

light travels $\frac{3}{10}\,m$ in 1 nsec

then $\quad 0 = -t\sinh\theta_r + x\cosh\theta_r$

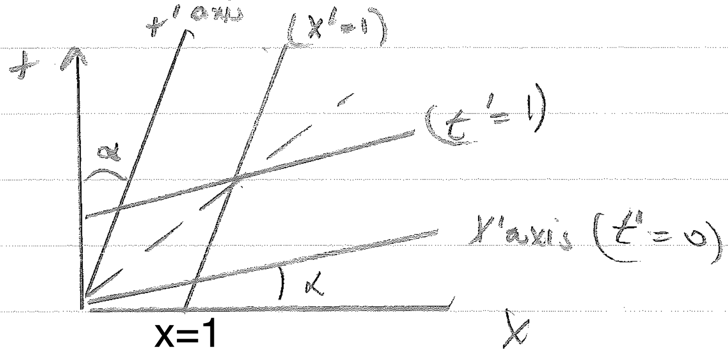
$\quad x = t\tanh\theta_r = \beta_r t \quad$ slope is $\beta_r = \tan\alpha$



lines of simultaneity of frame S' in S

$$\begin{pmatrix} 0 \\ x' \end{pmatrix} = \begin{pmatrix} \cosh\theta_r & -\sinh\theta_r \\ -\sinh\theta_r & \cosh\theta_r \end{pmatrix} \begin{pmatrix} t \\ x \end{pmatrix}$$

$t = \tanh(\theta_r)\, x = \beta_r x \qquad \beta_r = \tan\alpha$



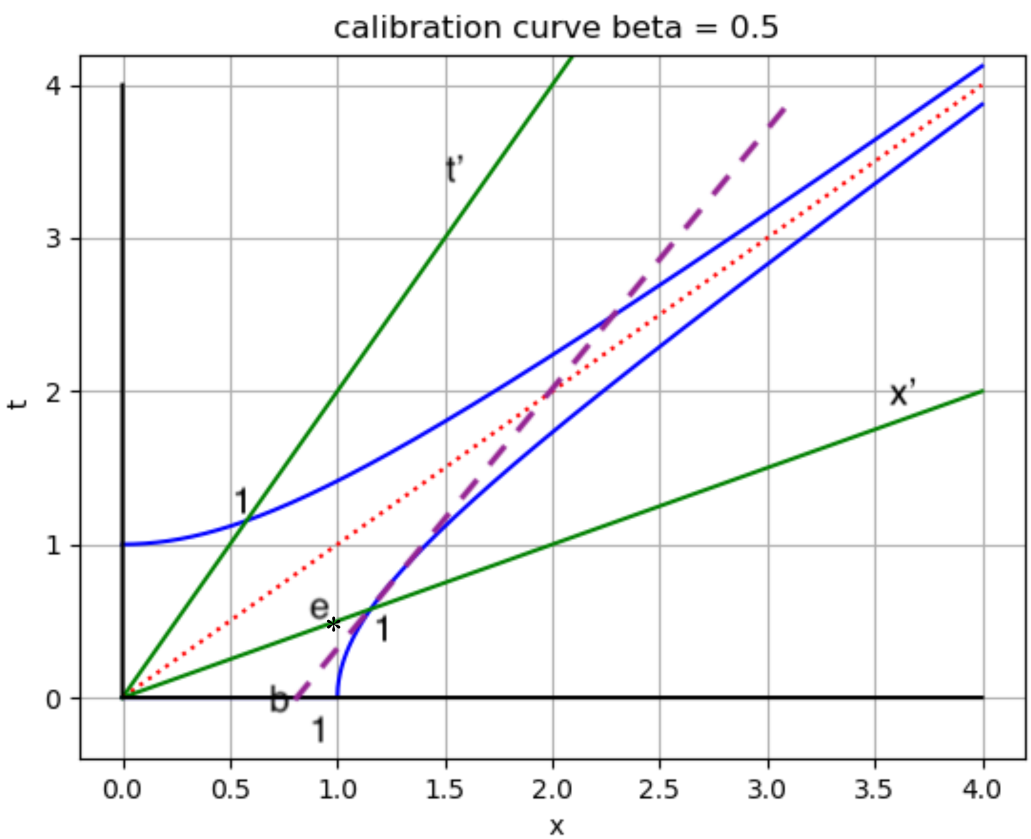
lines of constant x' & simultaneity lines cross along diagonal - Both S, S' measure speed of light pulse from the origin at $t=t'=0$ as having ~~speed~~ 1 (speed of light)

calibration lines - axis are stretched.

time calibration: $t^2 - x^2 = t'^2 - x'^2 = 1$

space calibration $x'^2 - t'^2 = x^2 - t^2 = 1$

Date: December 17, 2019 at 4:43 PM
Topic: spacetime



point e shows meter stick at rest is S is measured Lorentz contracted in S'.
point b shows meter stick at rest is S' is measured Lorentz contracted in S.
dashed line is parallel to t' axis and tangent to calibration curve.

blue calibration lines:

tcal: t(x) = sqrt(x^2+1)

xcal: t(x) = sqrt(x^2-1)

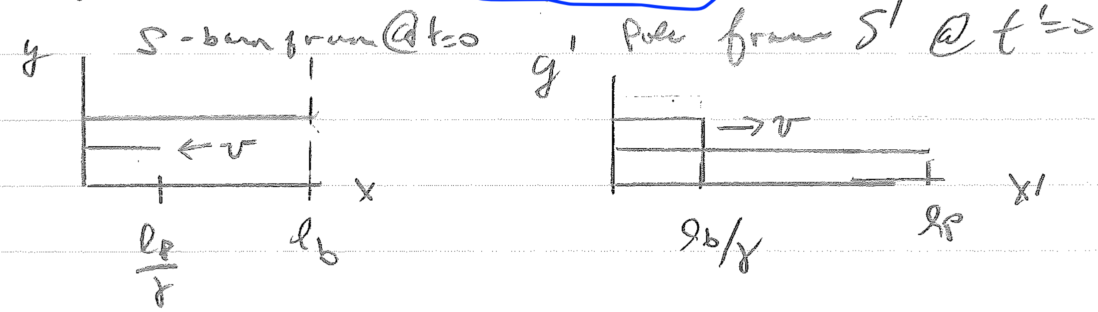
e: meter stick at rest in S is contracted in S'

b: meter stick at rest in S' is contracted in S

$\boxed{\text{Pole and Barn paradox variaton}}$ pole enters barn from left, back door of barn on right is closed.

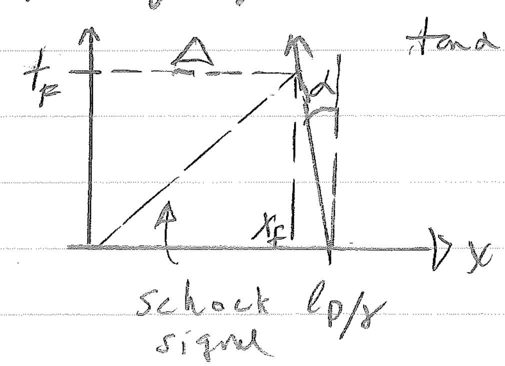
Proper length pole $= l_p$, barn $= l_b$ with $l_p > l_b$
pole moves in $-x$ direction with speed v such that $\frac{l_p}{\gamma} < l_b$. Take $t = t' = 0$, $x = x' = 0$ when

front of pole strikes back of barn.



Does pole fit in barn? Solution - No perfectly rigid objects

In frame S, back of pole continues to move until shock wave signal arrives at back of pole from front. Fastest this signal can travel is speed of light.



$\tan\alpha = v$

world line of back of pole

$x^{BP}(t) = \frac{l_p}{\gamma} - vt$

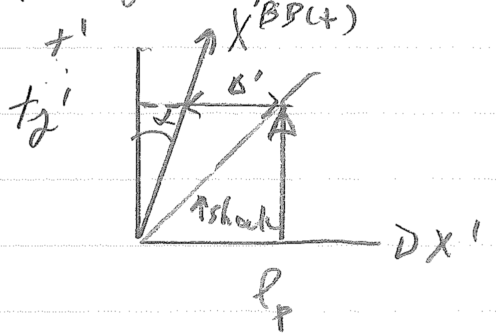
shock signal

$x^{sh}(t) = t$

$$x^{BP}(t_f) = x^{sh}(t_f) \implies t_f = \frac{l_p}{\gamma}\left(\frac{1}{v+1}\right)$$

$$\Delta = x_f = \frac{l_p}{\gamma} - v\frac{l_p}{\gamma}\frac{1}{v+1} = \frac{l_p}{\gamma}\left(\frac{1}{1+v}\right)$$

In pole frame, back end of barn $\overset{BB}{}$ crushes pole



world line of back of barn

$$x'^{BB}(t') = vt'$$

$$t_f' = \frac{\ell_p}{1} \quad \text{(speed of light} = 1)$$

$$x'^{BB}(t_f') = v\ell_p$$

$$\Delta' = \ell_p - x^{BB'}(t_f') = \ell_p(1-v)$$

check $\Delta' = \frac{\Delta}{\gamma} = \frac{\ell_p}{\gamma^2}\left(\frac{1}{1+v}\right) = \ell_p \frac{1-v^2}{1+v} = \ell_p(1-v)$

Also $\tilde{x}_F = \begin{pmatrix} t_f' \\ x_f' \end{pmatrix} = \ell_p \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

$$= \begin{pmatrix} t_f \\ x_f \end{pmatrix} = \frac{\ell_p}{\gamma}\left(\frac{1}{1+v}\right)\begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} t_f' \\ x_f' \end{pmatrix} = \underbrace{\gamma\begin{pmatrix} 1 & v \\ v & 1 \end{pmatrix}}_{\substack{L.T. \\ S \to S'}} \underbrace{\left(\frac{\ell_p}{\gamma}\,\frac{1}{1+v}\right)\begin{pmatrix} 1 \\ 1 \end{pmatrix}}_{\tilde{x}_F} = \ell_p \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$