phys 330 Pandemic Spring 2020

Lec #20 ; A Bit of Cosmology

Basic ideas of General Relativity (GR)

Einstein circa 1905

Motion in arbitrary (non-inertial, accelerated) frames.

Gravity and Inertia - Why should gravity couple to inertial mass?

law of inertia: $\vec{F} = m_I \ddot{\vec{r}}$

Newton's universal law of Gravitation $F_G = G \frac{m_g^{(1)} m_g^{(2)}}{r_{12}^2}$

Compare to Coulomb $F_c = k \frac{q_1 q_2}{r_{12}^2}$

From experiment, we know that gravitational "charge" m_g is identical to inertial mass.

$$\cancel{m}_I |\ddot{\vec{r}}| = \cancel{m}_g \frac{G m_g^E}{R_E^2} \qquad m_I = m_g$$

All objects fall with same acceleration due to gravity.

Weak principle of equivalence.

Torsion Balance (Eötvos experiment) e.g. berillium, titanium

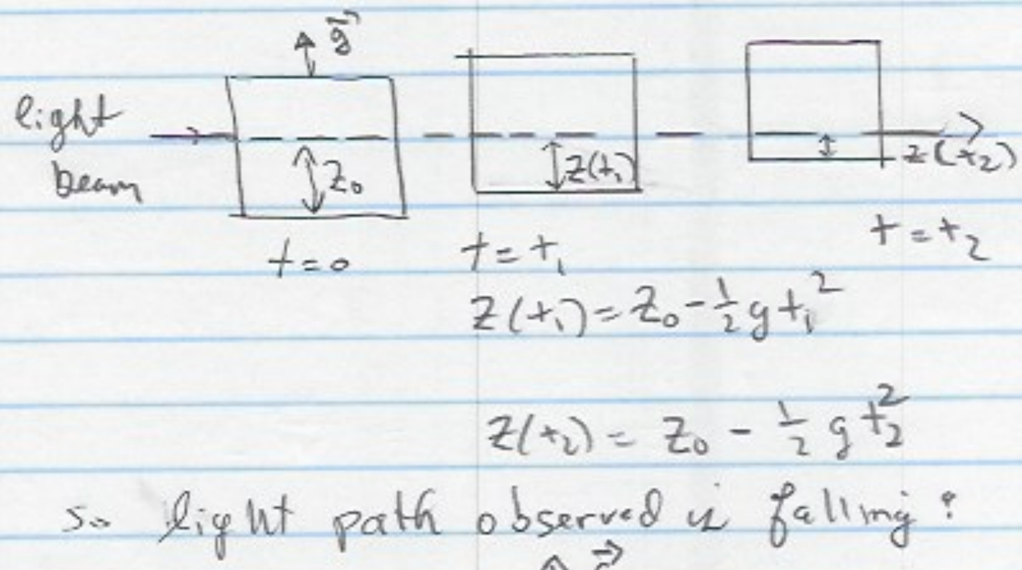
Eötvos parameter $\eta(Be-Ti) = (0.3 \pm 1.8) \times 10^{-13}$ (2012)

U.W. group $\eta = \Delta a / \left(\frac{a_1 + a_2}{2}\right)$ $\Delta a(Be, Ti) \lesssim 10^{-15}\ m/s^2$

Strong principle of equivalence: No experiment can distinguish between a uniform gravitational field and a uniformly accelerated reference frame.

Each observer sees same acceleration of dropped ball.

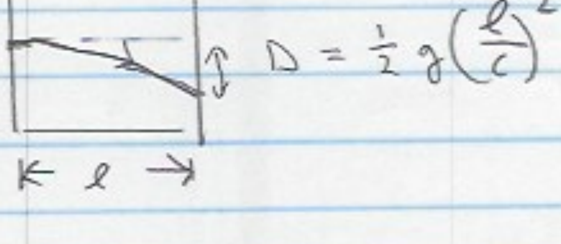
Predicts: ① Bending of light by gravity



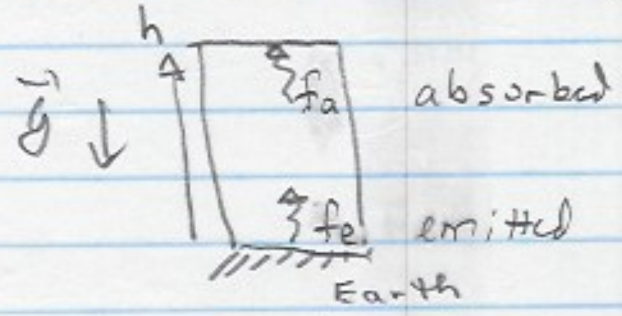
$$z(t_1) = z_0 - \frac{1}{2} g t_1^2$$

$$z(t_2) = z_0 - \frac{1}{2} g t_2^2$$

so light path observed is falling:



② Gravitational Red shift



Can also be thought of as photon energy decreasing due to potential energy

$$E_a = E_e - \left(\frac{E_e}{c^2}\right) g h$$

← gravity couples to energies in GR

"Apparent weight of photons"

$$\frac{E_a}{E_e} = \frac{f_a}{f_e} = 1 - \frac{gh}{c^2}$$

$$\frac{\Delta f}{f_e} = \frac{f_a - f_e}{f_e} = -\frac{gh}{c^2}$$

for $l = 100\,m$ $\quad \frac{\Delta f}{f} = \frac{(10\,m/s^2)\,100\,m}{(3\times10^8\,m/s)^2} = \underline{\underline{10^{-14}}}$

Measured by Pound & Rebka (1959) shortly after discovery of Mössbauer effect.

Mössbaur effect

$^{57}Co \rightarrow {}^{57}Fe^{*} + \nu_e$

K capture - inner electron $e^- + p \rightarrow n + \nu_e$

$^{57}Fe^{*} \rightarrow {}^{57}Fe + \gamma \qquad \tau = 10^{-7}\,s$

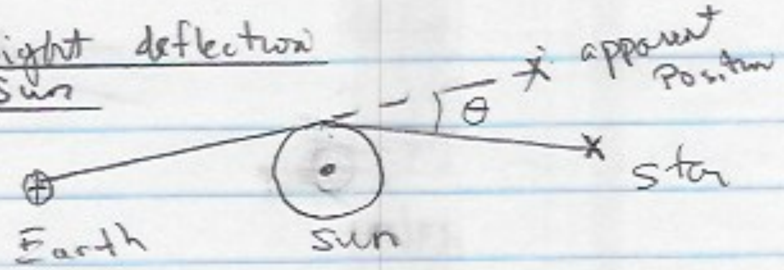
$$\Delta = E^{*} - E = 14.4\,keV$$

You will show $E_\gamma = \Delta - \epsilon$
due to nuclear recoil.
ε is not small compared to
natural line width.

Mössbaur discovered that emitted γ
could be resonantly absorbed when
Co nucleus was embedded in iron.
Co electroplated onto iron, then heated to
diffuse Co into iron. Fe^{*} embedded in
iron crystal lattice does not recoil.

Pound & Rebka (1959) $\dfrac{\Delta f^{exp.}}{\Delta f^{theory}} = 1.05 \pm 0.1$

Starlight deflection by Sun

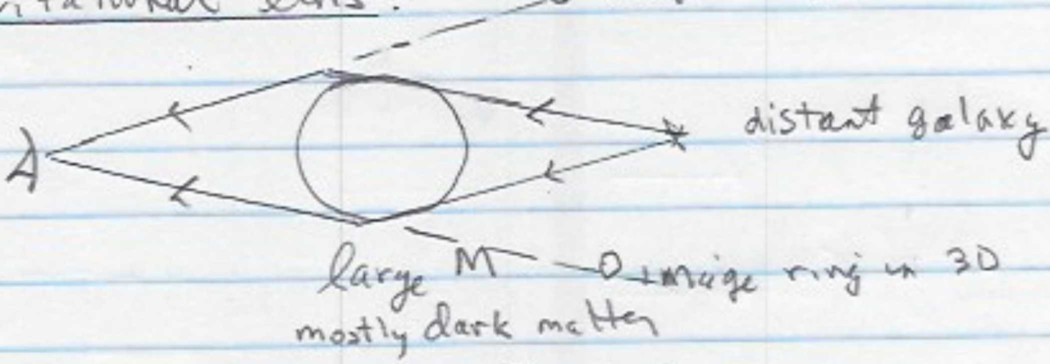


Eddigton, Solar eclipse 1919

$$\theta = \frac{4 M_\odot G}{R_\odot c^2} = 1.75'' = 1.75\left(\frac{1}{60}\right)^2 \frac{2\pi}{360}$$

$$= 8.5\times10^{-6}\ \underline{\text{radians}}$$

Gravitational lens:



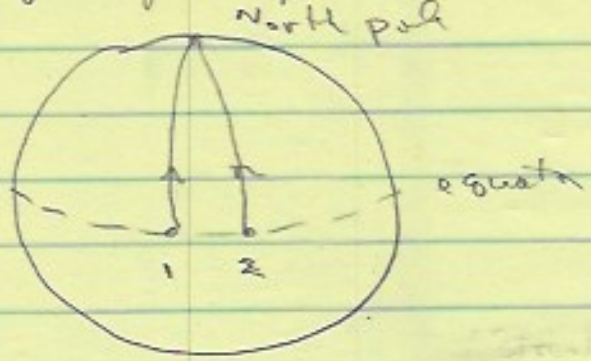
star will appear as Einstein ring.

Seen with Hubble telescope.

Force & Geometry in GR

Curvature of spacetime gives rise to "apparent" force. Particles follow straight line trajectories in curved spacetime.

A 2D analogy - Universe curved in space as the surface of a sphere.



straight lines are geodesics (great circles) intersections of plane through the center of sphere with surface of sphere. Curves 1 and 2 get closer together as they move along straight lines as if attracted by a force.

Line element on sphere of radius R:

$$(ds)^2 = (R\sin\theta\, d\phi)^2 + (R\, d\theta)^2$$

$$ds = R\left[(\sin\theta\, d\phi)^2 + (d\theta)^2\right]^{1/2}$$

ϕ azimuth angle

θ polar angle

Line element in flat spacetime

$$-(ds)^2 = +(d\tau)^2 = \underbrace{(c\,dt)^2}_{\equiv (dx_0)^2} - (dx_1)^2 - (dx_2)^2 - (dx_3)^2$$

shortest distance is path of light ray

$$d\tau = 0$$

Curved spacetime, path of light ray is "Geodesic"

$$(ds)^2 = \sum_{\mu,\nu=0}^{3} g_{\mu\nu}(t,\vec{x})\, dx^\mu dx^\nu$$

$$\bar{x} = (t, \vec{x}) = (x^0, x^1, x^2, x^3) \quad c = 1$$

$g_{\mu\nu}(t,\vec{x}) = g_{\mu\nu}(\bar{x})$ metric tensor

metric tensor is 4x4 matrix of function of $\bar{x}$.

Einstein Field equations

$$G_{\mu\nu}\left(g_{\mu\nu}(\bar{x})\right) = \frac{8\pi G_N}{c^4}\, T_{\mu\nu}(\bar{x})$$

G_N is Newton's constant

$G_{\mu\nu}, T_{\mu\nu}, g_{\mu\nu}$ are tensors transforming

in a certain way under $\bar{x} \to \bar{x}'$ coordinate change.

$G_{\mu\nu}$ describes geometry, depends on
derivatives of $g_{\mu\nu}$

$$\frac{\partial^2 g_{\alpha\beta}}{\partial x^\mu \partial x^\nu} \quad \text{e.g.} \quad \frac{\partial^2 g_{00}}{\partial x_1 \partial x_2}$$

$T_{\mu\nu}$ desribes mass-energy density
called stress-energy tensor.

16 coupled second order, non-linear
P.D.E. for $g_{\mu\nu}(\bar{x})$

"geometry = mass/energy density"

lec 20-9

Cosmological constant

To allow static universe, Einstein added the cosmological constant (Λ) -

$$G_{\mu\nu} - g_{\mu\nu}\Lambda = \frac{8\pi G}{c^4} T_{\mu\nu}$$

dimensionally energy density

moved to right hand side of equation, interpreted as vacuum energy.

No reason to exclude this term by symmetry of G.R. ($g_{\mu\nu}$ is a tensor).
In this sense, it was a mistake to leave it out.

For $\Lambda > 0$, acts as a cosmic scale repulsive force. measured to be small but not zero.

Vacuum energy in Q.F.T. diverges.

$$\langle E_{vac} \rangle = \sum_{\text{all frequencies}} \tfrac{1}{2} hf \rightarrow \infty$$

non-zero ground state of harmonic oscillator.

If we assume some high frequency cut-off f_c where Q.F.T. breaks down

$$\langle E_{vac} \rangle = \frac{U}{V} = \int \frac{d^3p}{h^3} E = \frac{4\pi}{(hc)^3} \int_0^{hf_c} E^3 dE = \frac{4\pi}{(hc)^3} (hf_c)^4$$

$$\Lambda = 0.73 \times 10^{-29}\, \mathrm{gm/cm^3} \sim (10^{-11}\, \mathrm{GeV})^4$$

measured

$\boxed{\text{Cosmological constant problem}}$

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<u>Cosmology</u> On very large scales universe is homogeneous and isotropic.

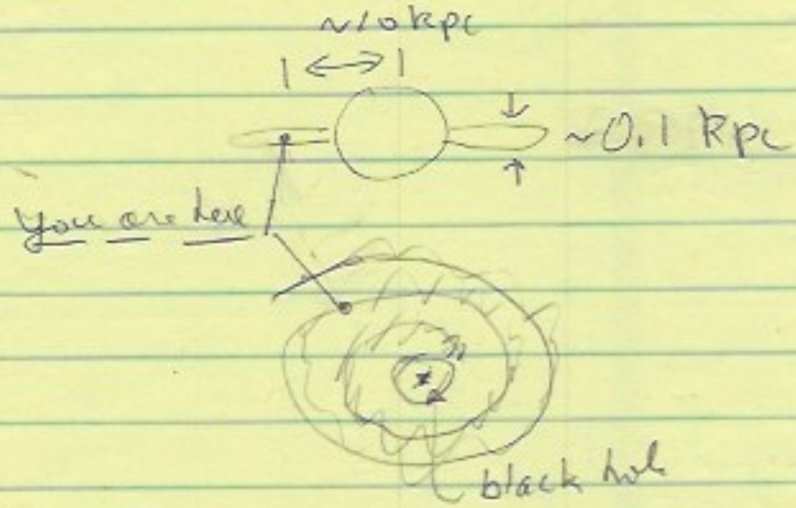
Cosmological scale $d \sim 100\,Mpc = 10^8\,pc$ (parsec)

$pc = 3.26\,ly$, $60\,Mpc \cong 10^8\,ly$ (light-year)

$1\,ly = 0.946 \times 10^{13}\,km$ $\quad \underline{1\,Mpc = 3 \times 10^{19}\,km}$

$\frac{\delta \rho}{\rho} \sim \frac{1}{10}$ from observation of <u>peculiar</u> <u>velocity</u> velocities of galaxies (expansion removed).

Hubble 1929 First to observe galaxies outside milky way



distance to Sun $6 \times 10^{-6}\,pc$

distance to nearest star ($10^4 \times$ distance to Pluto) $= 1\,pc$

nearest galaxy Andromeda $725\,Kpc$

edge of visible universe $3 \times 10^{10}\,pc$

$= 300\,d$

Isotropic, homogeneous universe gives

$T_{\mu\nu}$ diagonal.

ρ = energy density

$p \equiv -\frac{\partial E}{\partial V}$ pressure

$$T_{\mu\nu} = \begin{bmatrix} \rho & & & \\ & -p & & 0 \\ & 0 & -p & \\ & & & -p \end{bmatrix}$$

$$\equiv \text{diagonal}(\rho, -p, -p, -p)$$

with $t_0 \equiv$ now

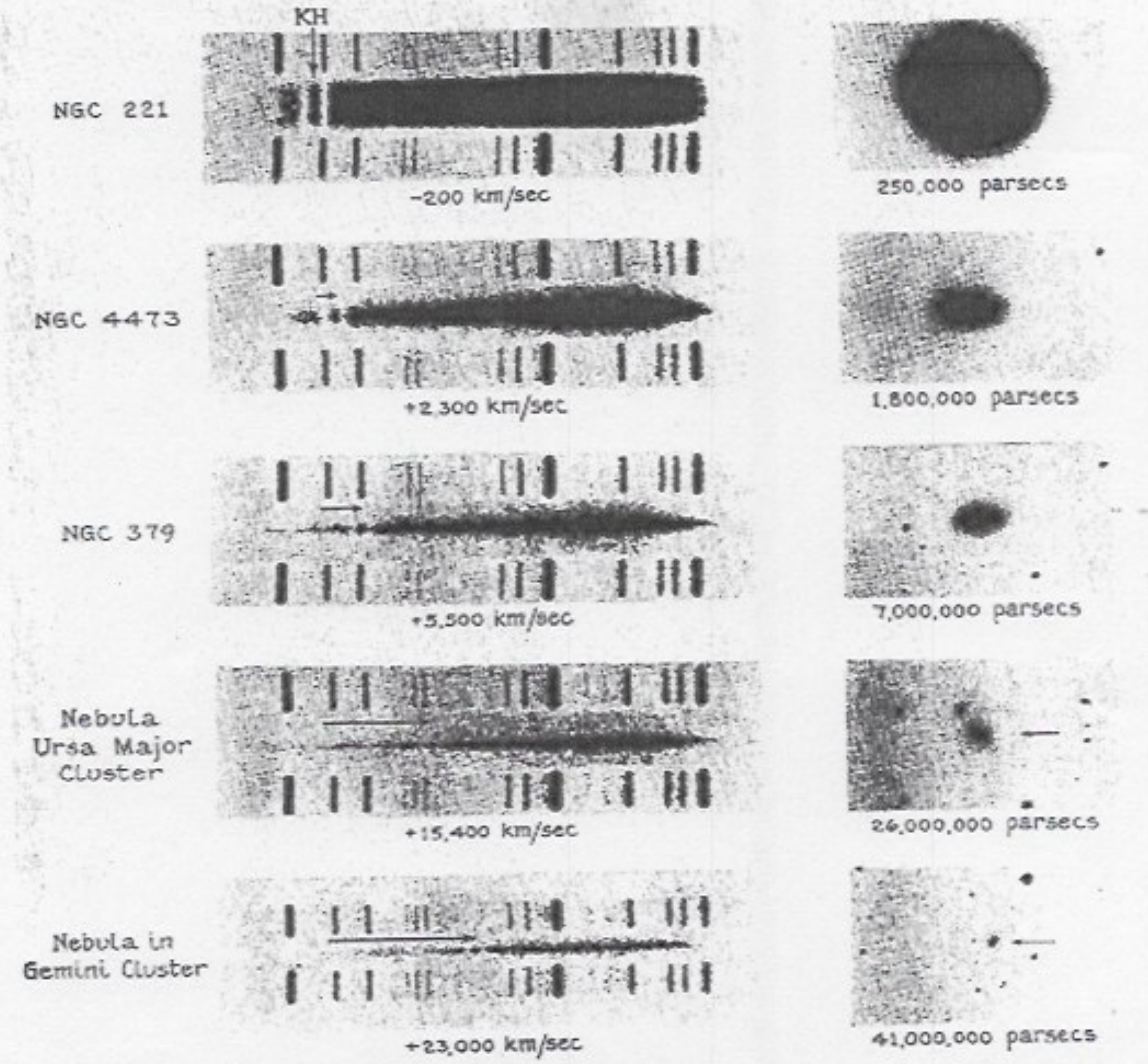
$$\rho_m(t_0) = 4.6 \times 10^{-33}\ g/cm^3$$

$$= 4.6 \times 10^{-27}\ g/m^3$$

$$\cong (1\,\text{proton})/m^3$$

Expanding Universe -

Hubble discovered "distant stars" actually galaxies (1929). Measured velocity red shift.



red shift

$$z \equiv \frac{\lambda_{obs} - \lambda_{emit}}{\lambda_{emit}} = \frac{\lambda_{obs}}{\lambda_{emit}} - 1 = \frac{V_r}{c}$$

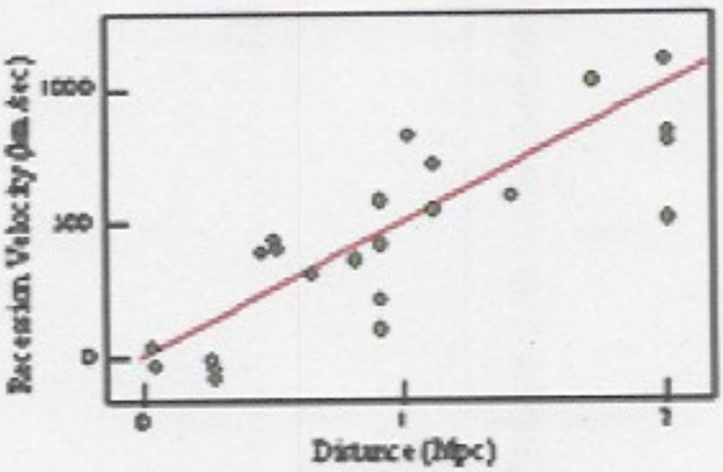
V_r = doppler shift recession velocity

Red shift due to expanding universe interpreted as recession velocity.

$$z_{max} = \frac{V_r^{max}}{c} = \frac{10^3 \text{ km/s}}{3\times10^5 \text{ km/s}} = 0.03$$

Hubble's Data (1929)

V_r (km/s)



$v_r = H_0 d$

slope is Hubble Constant

d = distance Mpc

measured today:

$$H_0 = 100h \text{ km/(Mpc}\cdot\text{s)}$$ inverse time

$$h = (0.682 \pm .006)$$ (model dependent)

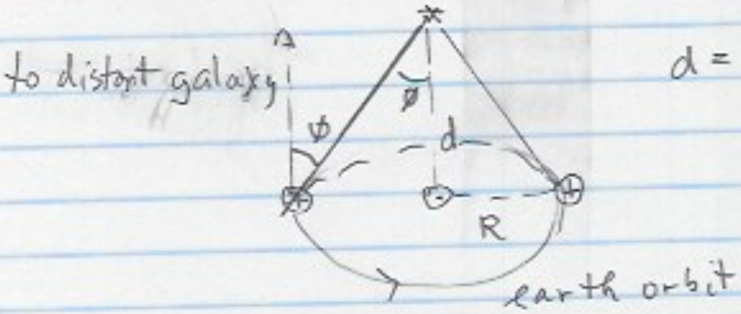
Hubble (expansion time)

$$\tau_H = \frac{1}{H_0} = \frac{1}{70}\, s \underbrace{\left(\frac{\text{Mpc}}{\text{km}}\right)}_{=3\times10^{19}} = \frac{1}{0.7}\, 3\times10^{17}\, s = 4\times10^{17}\, s$$

$$\underline{\tau_H = 1.4\times10^{10}\, y = 14 \text{ billion years}}$$

Distance measure

1) Parallax: apparent motion of stars relative to very distant galaxies ("fixed stars")



$$d = \frac{R}{\tan\phi} \simeq \frac{R}{\phi}$$

example: $\phi = 10^{-6}$ radians $\quad d = \frac{1.5\times10^{8}\text{ km}}{10^{-6}} = 1.5\times10^{14}\text{ km}$

$$= 4.6\text{ pc}$$

2) Apparent brightness (ℓ) of "standard candle"

$$\ell = \frac{L}{4\pi d^2} \qquad L = \text{absolute luminosity}$$

$$\frac{\ell_{star}}{\ell_\odot} = \left(\frac{d_\odot}{d_{star}}\right)^2\Bigg|_{10\text{ pc}} = \left(\frac{6\times10^{-6}\text{ pc}}{10\text{ pc}}\right)^2 = 0.36\times10^{-14}$$

L depends on mass of star

"Main sequence" stars $\quad T \sim M \quad$ T temp, from Wein's displacement law
H fusion to helium
90% of stars

Dec 20 - 15

(3) Cepheid variables varying in intensity with period related to L

(4) type IA supernovae:
star accretes mass (white dwarf, star binary). When $m \sim 1.4\, M_\odot$ degeneracy pressure insufficient → Supernovae

method	max distance	
parrallax	30 p	
main sequence	0.1 MPC	
cepheid	4 MPC	
(brightest galaxies	few × 10 Gpc)	not standard candle
type IA	few × 10 Gpc	

Cosmological Models

$$T_{\mu\nu} = \text{diagonal}(\rho, -P, -P, -P)$$

Constant curvature, Robertson-Walker metric:

$$(ds)^2 = -c^2\, dt^2 + a(t)\left[\frac{dr^2}{1-kr^2} + r^2\left(d\theta^2 + \sin^2\theta\, d\phi^2\right)\right]$$

k is sign of curvature; a is "scale factor"

k	geometry	2D analog	Velocity analog*
+1	closed	sphere	$V < V_{esc}$
0	flat	plane	$V = V_{esc}$
−1	open	saddle	$V > V_{esc}$

* Velocity of object launched from earth surface

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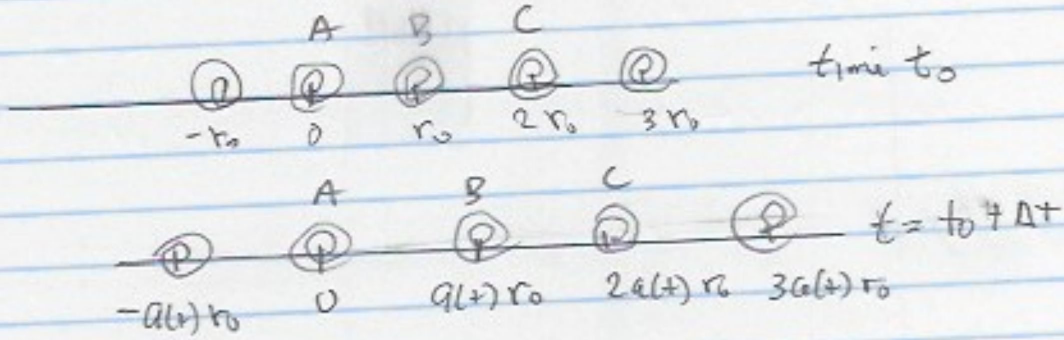
All three possibilities imply <u>big bang</u>.

<u>Expanding universe analogy</u>:
galaxies on rubber band



$$r(t) = r_0\, a(t)$$

$$\frac{\Delta r_{AB}}{\Delta t} = V_{AB} = r_0 \left[\frac{a(t+\Delta t) - a(t)}{\Delta t} \right] = r_0\, \dot{a}$$

$$\frac{\Delta R_{AC}}{\Delta t} = V_{AC} = r_0\, 2\dot{a}$$

Apparent recession speed $\propto$ distance

<u>Note</u> - big bang does not imply universe is spatially finite.

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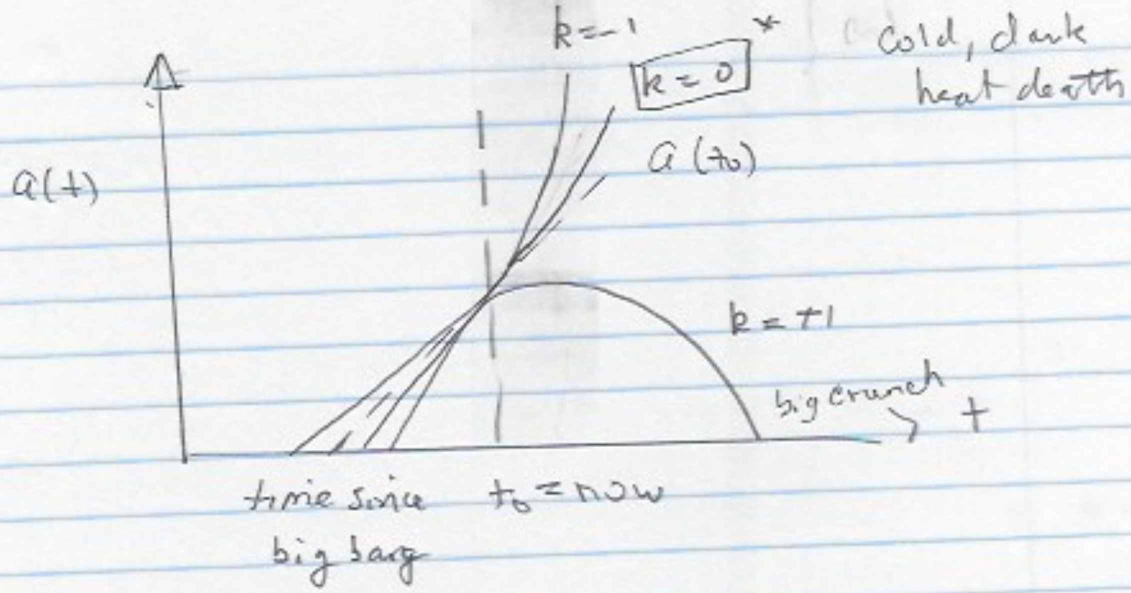

$$\frac{\dot{a}}{a} = H(t) \quad \text{with } H(t_0) \equiv H_0$$

three cosmologies drawn with same H_0.

time since big bang not H_0^{-1}. Depends not only on k, but evolution of a(t) which depends on (ρ, P).

<u>red shift</u>

$$1+z = \frac{\lambda_{obs}}{\lambda_{emit}} = \frac{a(t_0)}{a(t_e)}$$

directly measures scale factor

* <u>CMB observations strongly favor a spatially flat universe: $k=0$ and $\rho_{tot} = \rho_{crit}$.</u>

Critical density

Friedman equation

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{k}{a^2}$$

$\rho = \rho_m$ matter = baryons + dark

$+ \rho_r$ radiation

$+ \rho_\Lambda$ vacuum

Critical density for which $\underline{k=0}$ spatially flat universe.

$H_0 \equiv H(t_0)$

$$\rho_{crit} = \frac{3H_0^2}{8\pi G} = 1.88 \times 10^{-29} h^2 \ g/cm^3$$

$$h \equiv \left(\frac{H_0}{100 \ km/s/Mpc}\right) = 0.674$$

$$\rho_{crit} = 0.85 \times 10^{-29} \ g/cm^3 = 8.5 \times 10^{-27} \ kg/m^3$$
$$\hat{=} 5 \text{ protons}/m^3$$

$$= 1.26 \times 10^{11} \ M_\odot/(Mpc)^3$$

define $\rho(t_0)$ relative to ρ_{crit} as Ω

$$\Omega = \Omega_m + \Omega_r + \Omega_\Lambda = 1$$

for $k=0$ spatially flat universe

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Dyamics of Universe

Einstein equation + Robertson-Walker metric
(homogeneity, isotropy)

① $$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G\rho}{3} - \frac{k}{a^2} + \frac{\Lambda}{3} \quad (c=1)$$

pressure $p = -\frac{dE}{dV}$ E = energy

changing variables $\frac{dV}{dt} p = -\frac{dE}{dt}$

Volume $V = V_0 a^3(t)$

$E = V_0 a^3(t) \rho(t)$ V_0 const

$$p \frac{d}{dt}(a^3) = -\frac{d}{dt}(a^3\rho) = -\dot{\rho} a^3 - \rho \frac{d}{dt}(a^3)$$

$$a^3 \dot{\rho} = -(p+\rho)\frac{da^3}{dt} = -(p+\rho) 3a^2 \dot{a}$$

①′ $$\dot{\rho} = -(p+\rho) 3\left(\frac{\dot{a}}{a}\right) = -(p+\rho) 3H$$

Combine ①, ①′ with $k=0$ (take time derivative of 1)

② $$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p) + \frac{\Lambda}{3}$$

①, ② are Friedman equations

Modeling the Universe

On cosmic scale, universe is composed of perfect (zero viscosity) fluids, each described by an equation of state

$$P = w\rho \qquad w \text{ a constant}$$

recall ideal gas law,

$$P = \left(\frac{N}{V}\right) kT = \left(\frac{kT}{m}\right)\rho$$

Matter: galactic matter is a cosmic "dust" or pressureless gas ($w=0$)

$$\frac{d}{dt}\left(\rho_m a^3\right) = 0$$

$$\dot{\rho}_m a^3 + \rho_m 3 a^2 \dot{a}$$

$$\frac{\dot{\rho}_m}{\rho_m} = -3\frac{\dot{a}}{a} \qquad \text{use } \int_{t_0}^{t} \frac{\dot{x}}{x}dt = \int \frac{dx}{x} = \ln\frac{x(t)}{x(t_0)}$$

$$\ln\frac{\rho_m(t)}{\rho_m(t_0)} = \ln\left(\frac{a(t)}{a(t_0)}\right)^{-3}$$

$$\boxed{\rho_m(t) = \rho_m(t_0)\left(\frac{a(t_0)}{a(t)}\right)^3} \quad \text{matter} \quad \boxed{w=0}$$

Radiation: relativistic matter (e.g. ν), photons

$$P_\gamma = \frac{1}{3}\rho_\gamma \qquad \boxed{w = \frac{1}{3}}$$

$$\dot{\rho}_\gamma = -3H(\rho_\gamma + P_\gamma) = -4H\rho_\gamma$$

$$\frac{\dot{\rho}_\gamma}{\rho_\gamma} = -4\left(\frac{\dot{a}}{a}\right)$$

integrates to $\rho_\gamma(t) = \rho_\gamma(t_0)\left(\frac{a(t_0)}{a(t)}\right)^4$

since $\rho_\gamma = \frac{4}{c}\sigma T^4$ σ = Stephan-Boltzmann constant

$$\frac{\dot{\rho}_\gamma}{\rho_\gamma} = 4\frac{\dot{T}}{T} = -4\frac{\dot{a}}{a}$$

$$T(t) = T(t_0)\left(\frac{a(t_0)}{a(t)}\right) \qquad \boxed{T \propto \frac{1}{a}}$$

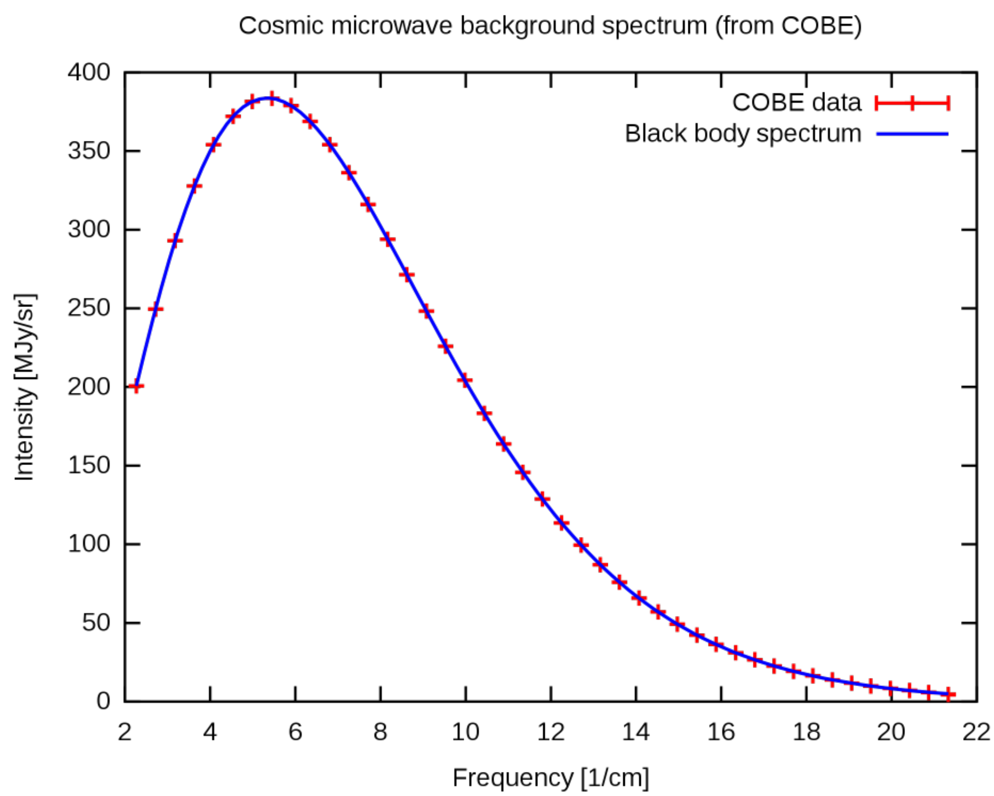
photon gas cools as universe expands

Since $\rho_m \propto \frac{1}{a^3}$ and $\rho_\gamma \propto \frac{1}{a^4}$

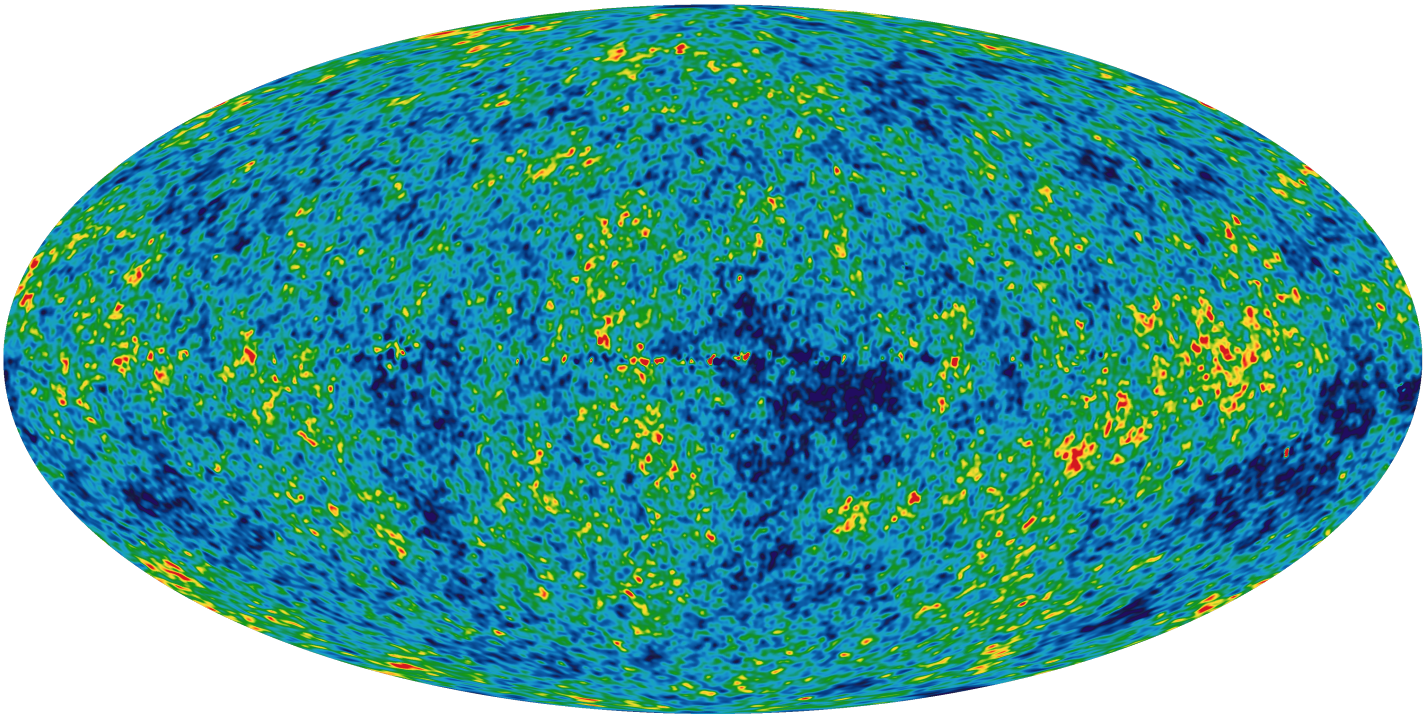
$$\frac{\rho_\gamma(t)}{\rho_m(t)} = \frac{\rho_\gamma(t_0)}{\rho_m(t_0)}\frac{a(t_0)}{a(t)}$$

Early universe was mostly hot radiation —

Hot big bang.



Observational error bars smaller than thickness of blue line. T_CMB = 2.7260±0.0013 K



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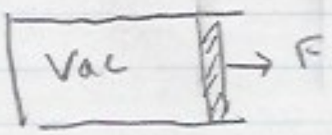
Cosmological constant Λ

Can be interpreted physically as vacuum energy density

Friedman equation ($k=0$)

$$\rho_{vac} = \frac{c^4}{8\pi G} \Lambda$$

$$P_{vac} = -\rho_{vac}$$



$$p = -\frac{dE}{dV} = -\rho_{vac}$$

A negative pressure or tension

Consider empty universe with just Λ.

$$\frac{\ddot{a}}{a} = \frac{-4\pi G_N}{3}\left[\rho_{vac} - 3(-\rho_{vac})\right] = \frac{8\pi G}{3}\rho_{vac}$$

$$= \frac{\Lambda}{3}$$

$$\ddot{a} = \frac{\Lambda}{3} a \Rightarrow a(t) = a(t_0) e^{\Lambda t/3}$$

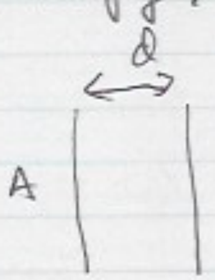
exponential expansion.

Λ acts like a cosmic scale repulsive force.

In QFT we saw that field ground state energy ($\frac{1}{2} h f_n$ per mode n) diverges.

If we ignore gravity, we are only interested in changes in energy, so we just throw it out because it is not measurable.

Experimentally, we can measure a change in vacuum energy – the Casimir effect



two grounded, electrically neutral conducting plates

lowest mode inside box is

$\lambda_0 = 2d$

$f_0 = \frac{c}{\lambda} = \frac{c}{2d}$

outside box, all modes exist
as d increases, energy in box increases

$\frac{\Delta E}{\Delta V} > 0$

Consistent with previous argument that vacuum has a tension

So two neutral, grounded conducting plates have an attractive force

$$F = 1.3 \times 10^{-7} \frac{N \cdot (cm)^4}{(cm)^2} \frac{A}{d^4}$$

crucial dependence on separation

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Precision

Measurement U. Mohideen et.al. (2000) using AFM
from hep-quant/0106045 review article

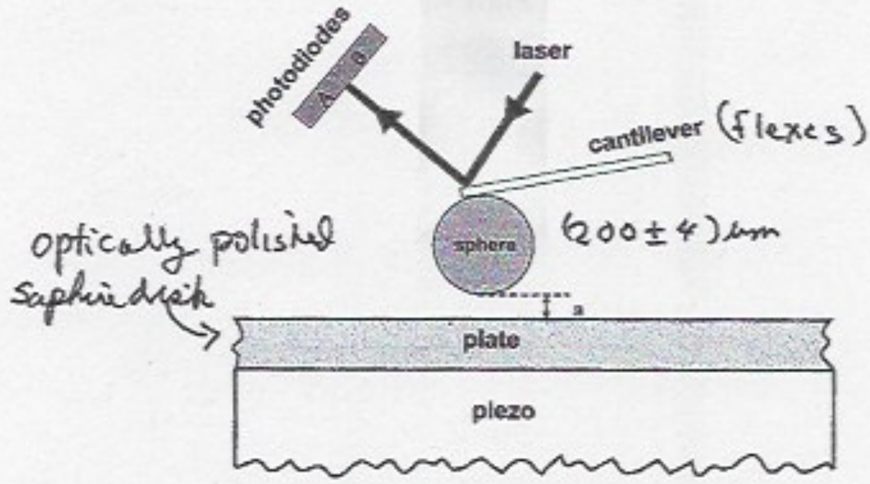

Fig. 14. Schematic diagram of the experimental setup. Application of voltage to the piezoelectric element results in the movement of the plate towards the sphere.

Concerns: ① smoothness of plate
② finite conductivity
③ contact potentials due to contact between metals with different work function
④ precision measurement of separation a.

experimental arrangement: atomic force microscope
polystyrene sphere attached w/ Ag epoxy
300 nm Al thermal evaporation
20 nm Au/Pd coating

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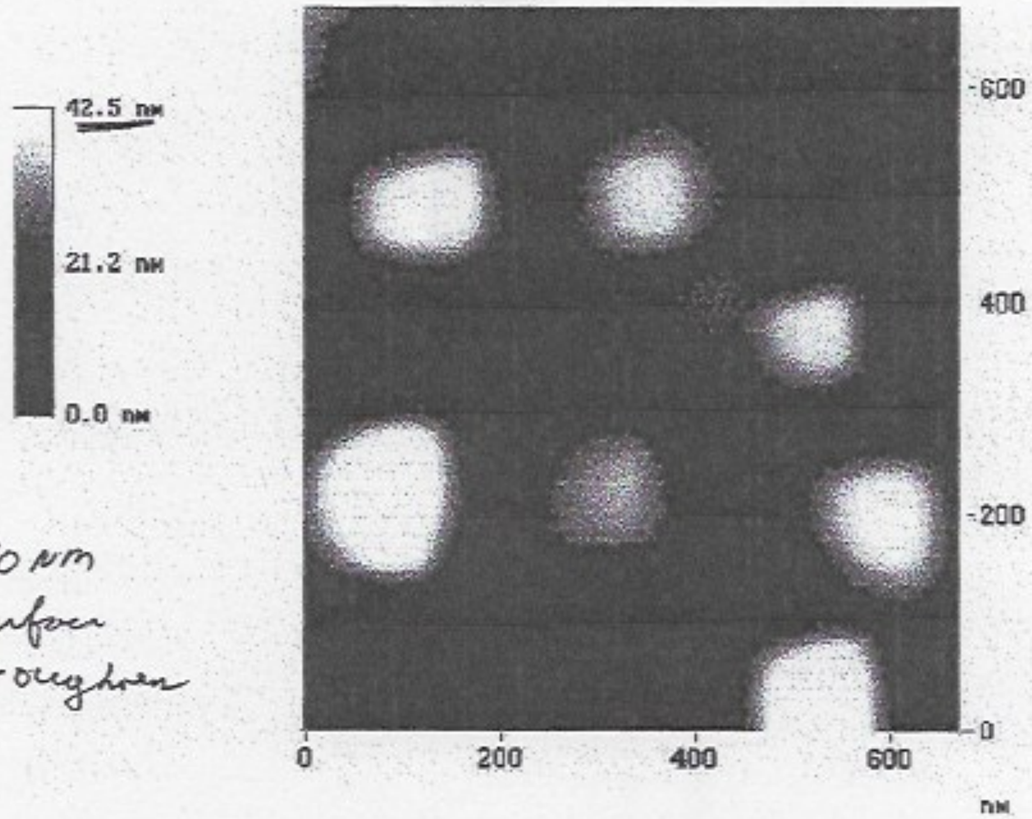

~40 nm
surface
roughness

Fig. 18. Typical atomic force microscope scan of the metal surface. The lighter tone corresponds to larger height as shown by the bar graph on the left.

measure surface roughness, conductivity
& calculate correction.

Dec 20-26

Experimental results: Force vs separation

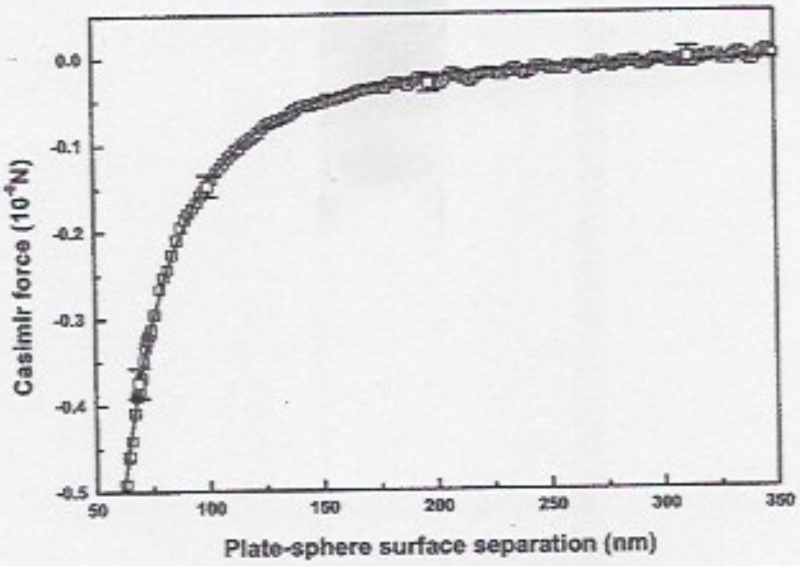

Fig. 24. The measured average Casimir force as a function of plate-sphere separation is shown as squares. For clarity only 10% of the experimental points are shown in the figure. The error bars represent the standard deviation from 30 scans. The solid line is the theoretical Casimir force with account of roughness and finite conductivity corrections.

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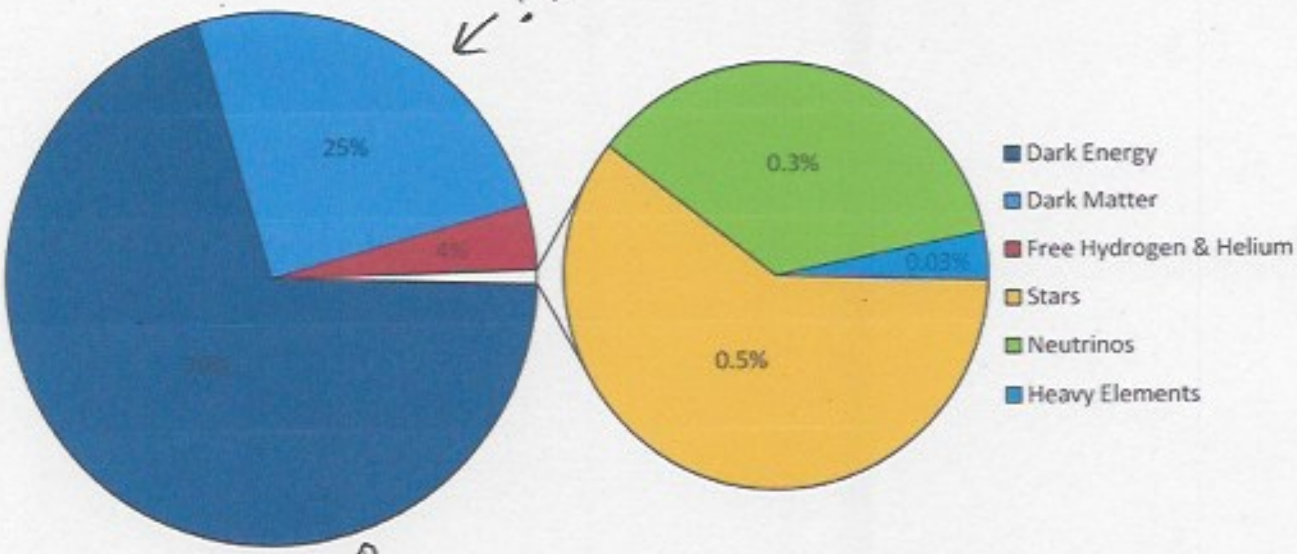
Content of universe today is precisely measured

? dark matter



? Λ vacuum energy

Inflation (Guth, 1979)

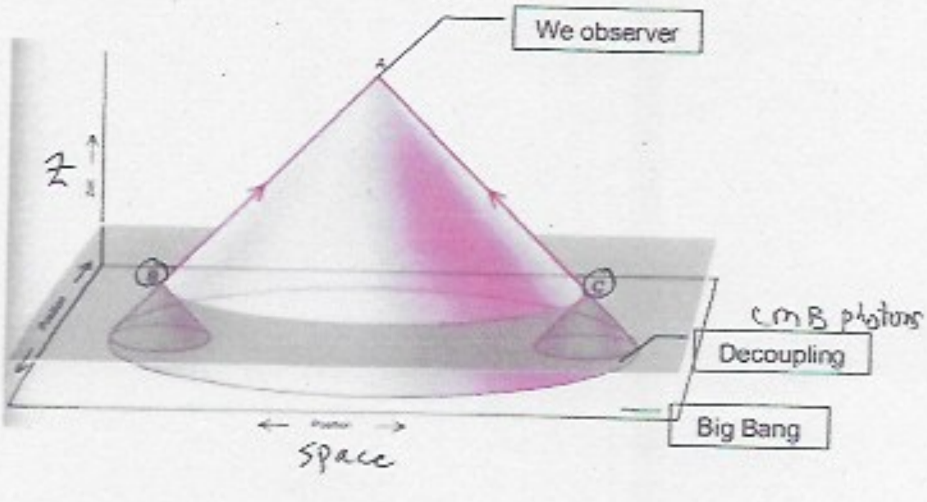
Theoretical idea motivated by

① absence of relic magnetic monopoles

② homogeneous, isotropic universe

③. spatially flat universe

④ horizon problem



patches B, C are causily disconected
but CMB is uniform in temperature
to 1/10,000 .

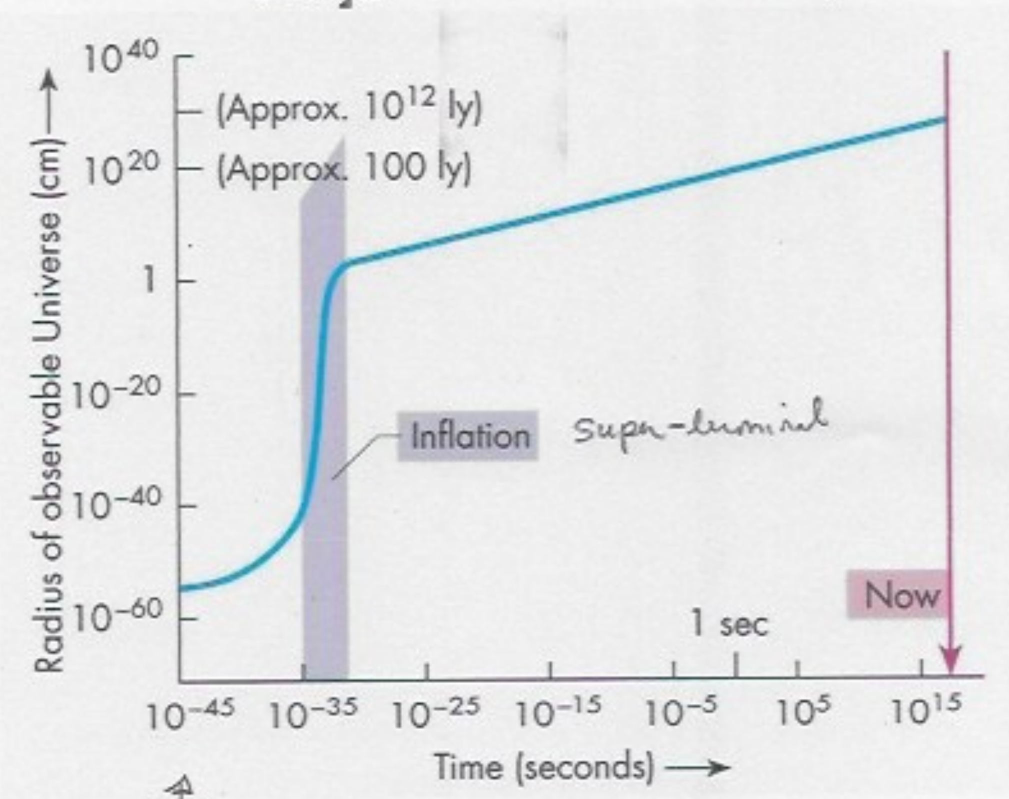
Lec 20-29

Postulate bosonic field called the <u>inflaton</u> that has non-zero expectation value in vacuum for $\sim 10^{-36}$ s. Universe expands exponentially.

$a \to \underline{\underline{10^{26}}}\, a$ proton $\to$ softball size.

inflating away any inhomogeneity, curvature

Vacuum $\xrightarrow[\text{decays}]{}$ physical inflaton $\xrightarrow[\text{decays}]{}$ matter, energy



Big Bang singularity

$\boxed{\text{predictions:}}$

① CMB fluctuations are scale invariant, Gaussian
seem to be

② CMB "B-mode" polarization
Big hunt is on. (talk to Prof. Barron)