Phys 330 Spring 2020

Lec #4 Origins of QM

① Planck and Black Body Spectrum

Universal spectrum of E.M. radiation from hole in cavity at temperature T.

$$T(\text{kelvin}) = T^{c}(\text{centigrade}) + 273.15K$$

$T=0$ absolute zero

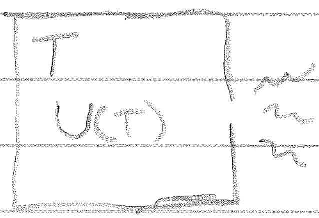
recall, monatomic ideal gas mean energy /molecule

$$\bar{\epsilon} = \frac{3}{2}kT$$

k is Boltzmann Constant

$$kT\Big|_{300K} \simeq \left(\frac{1}{40}\right)eV$$

300K room temperature



enclosure @ temp T

small hole measure radiation from hole

or $\left[\frac{\text{Power}}{\text{area (wavelength)}}\right] = \frac{dR}{d\lambda} = \frac{\text{radiated energy}}{\text{time} \cdot \text{area} \cdot \text{wavelength}}$

$$\frac{dR}{d\lambda} = \frac{c}{4}\frac{dU}{d\lambda}$$

U = energy/volume at T of radiation

Change variable: $hf = hc/\lambda \equiv E$

$\frac{dU}{d\lambda} d\lambda = \frac{dU}{dE} dE = u(E)\, dE$

lower case u

Plack guessed (and later derived) universal form,

$$u(E) = 8\pi \left(\frac{kT}{hc}\right)^3 \frac{(hf/kT)^3}{(e^{hf/kT} - 1)}$$

where Planck introduced constant h evaluated from fit to data (circa 1900) now known precisely

$h = 6.626 \times 10^{-34}$ J·s $\quad hc \cong 1240$ eV·nm

$\hbar \equiv \frac{h}{2\pi}$, $\hbar c = 197$ eV·nm

$u(E)$ is number density of photons

$\int_0^\infty u(E)\, dE = U = \frac{\text{energy}}{\text{vol}}$ of radiation

of photon gas in equilibrium @ T.

Planck's derivation assumed energy emitted/absorbed in quanta of energy,

$\varepsilon = hf.$

4-3

Modern Physics 330 *January 11, 2020*

Black Body Spectrum

Photon gas energy density $u(E)$ at temperature T, where photon energy E=hf, f being the frequency. The energy density of the gas between E and $E + dE$ is $u(E)dE$. The function $u(E)$ has dimensions of inverse volume and depends on dimensionless variable $x = \frac{E}{kT} = \frac{hf}{kT}$.

$$u(E) = 8\pi \left(\frac{kT}{hc}\right)^3 \frac{(\frac{E}{kT})^3}{e^{\frac{E}{kT}} - 1}$$

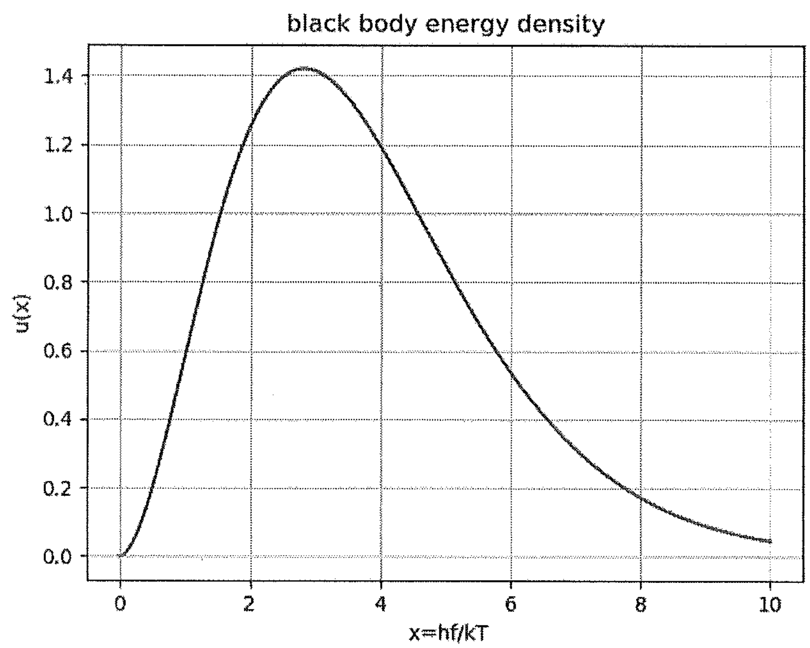


Figure 1: Plot of $\frac{x^3}{e^x-1}$: peak value 1.421 at $x = \frac{hf}{kT} = 2.821$

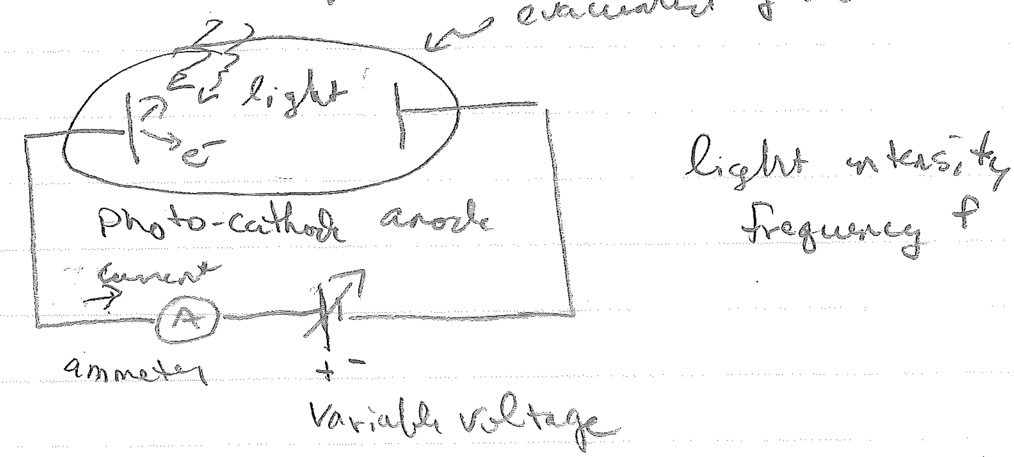
Wein's displacement law for shift of peak wavelength with temperature T:

$$\lambda_{peak} = \frac{b}{T}$$

where constant $b = 2.898 \times 10^{-3} \text{m} \cdot \text{K}$

② Einstein and the Photon (1905)

Photo-Electric effect.



light intensity I,
frequency f

Photo current goes to zero at some "stopping" voltage V_s corresponding to max. K.E. of electrons $E_s = eV_s$

observe –

① no photo electrons if incident light has $f < f_{min}$.

② f_{min} depends on photo-cathode metal

③ if $f > f_{min}$, photo-electrons emitted instantly regardless of I

④ Photo-current increases with I but independent of f

⑤ E_s independent of I but depends on f

<u>Impossible</u> to explain from classical E&M.

Maxwell EM wave energy flux

$\vec{S} = c^2 \varepsilon_0 \vec{E} \times \vec{B}$ Poynting vector

note on units: $\vec{E} = \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2} \hat{r}$ charge q

$\vec{B} = \frac{\mu_0 i}{2\pi r} \hat{\phi}$ current $i = \frac{dq}{dt}$

$\varepsilon_0 = \frac{1}{\mu_0 c^2}$ $c \cong 3 \times 10^8$ m/s

$\mu_0 = 4\pi \times 10^{-7}\ N A^{-2}$ Newton N

Ampere A

$\varepsilon_0 e^2 = 1.438$ eV.nm e charge on electron

Coulomb force constant

light intensity I $\left[\frac{\text{energy}}{\text{area}\cdot\text{time}}\right]$

$I = c\varepsilon_0 \langle \vec{\mathcal{E}} \cdot \vec{\mathcal{E}} \rangle_{\text{time average}}$

For plane wave of frequency f, $\lambda = \frac{c}{f}$

$k = 2\pi/\lambda$ $\omega = 2\pi f$

$\vec{E} = \vec{E_0} \cos(kx - \omega t)$

$|\vec{E_0}|$ is electric field amplitude

$$\langle \vec{E}\cdot\vec{E} \rangle = \frac{1}{2} E_0^2$$

$$I = \frac{c\,\varepsilon_0}{2} E_0^2 \quad (\text{Amplitude})^2$$

independent of f.

<u>most suprising</u> at the time was immediate appearance of photo-current.

Classical EM predicts a long time —

Power = I·(area)

energy = I·area·time > W, work function of metal

$W \simeq 4\,eV$

to liberate an electron from an atom

$$\text{area} = \left(\tfrac{1}{3}\,nm\right)^2$$

For $I = \dfrac{1\,mW}{m^2} = 10^{-3}\,\dfrac{J}{s\,m^2}$

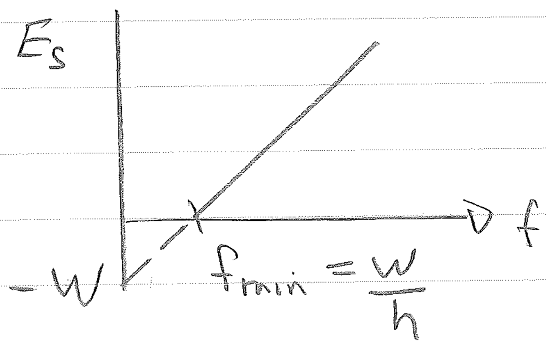
$$= \frac{10^{-3}\ J/(s\,m^2)}{1.6\times10^{-19}\ J/eV} = 6.25\times10^{15}\ eV/m^2$$

$$= 6.25\times10^{-3}\ eV/(nm)^2$$

$$\text{time} > \frac{\Phi}{I\,\text{area}} = 4eV\ \frac{1}{6.25\times10^{-3}\ eV/nm^2}\cdot\frac{1}{\left(\frac{1}{3}nm\right)^2}$$

$$= 6\times10^{3}\ s = 100 \text{ minutes.}$$

Einstein's elegant solution
light energy in photon $\varepsilon = hf$
using Planck's constant

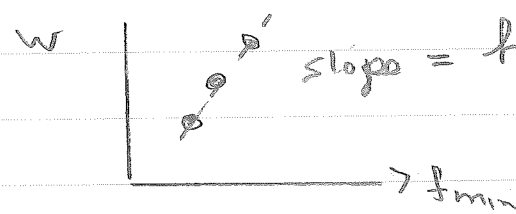


$$hf = W + E_k$$

E_k is kinetic energy

$$hf_{min} = W$$

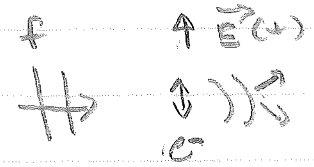
measure f_{min} vs. W for different materials of known W.



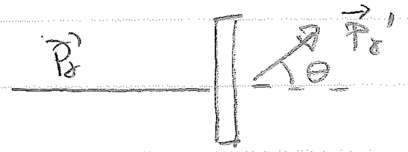
material	W(eV)
Al	4.08
Cu	4.70
Ag	4.73

Compton Scattering (1923)

Classical EM scattering:



charge oscillates at same f as light
scattered light has same f



pb target

e⁻ binding energy negligible.

$\gamma + e^- \to \gamma + e^-$ with e^- initially at rest.

$$\bar{P}_{TOT} = \begin{pmatrix} P_\gamma + m \\ P_\gamma \hat{x} \end{pmatrix} = \begin{pmatrix} P_\gamma' + E \\ \vec{P}_\gamma' + \vec{P}_e \end{pmatrix}$$ 4-momentum conservation

$$P_\gamma + m = P_\gamma' + \sqrt{P_e^2 + m^2}$$

$$P_e^2 = |\vec{P}_e|^2 = |\vec{P}_\gamma' - P_\gamma \hat{x}|^2$$

$$= P_\gamma'^2 - 2P_\gamma' P_\gamma \cos\theta + P_\gamma^2$$

and $P_e^2 + m^2 = (P_\gamma - P_\gamma' + m)^2$

$$(P_\gamma - P_\gamma')^2 + 2m(P_\gamma - P_\gamma') + m^2$$

eliminate P_e –

$$(P_\gamma - P_\gamma')^2 + 2m(P_\gamma - P_\gamma') = P_\gamma'^2 - 2P_\gamma P_\gamma' \cos\theta + P_\gamma^2$$

$$-2P_\gamma P_\gamma' + 2m(P_\gamma - P_\gamma') = -2P_\gamma P_\gamma' \cos\theta$$

$$m(P_\gamma' - P_\gamma) = P_\gamma P_\gamma' (1 - \cos\theta)$$

$$\left(\frac{1}{P_\gamma'} - \frac{1}{P_\gamma}\right) = \frac{1}{m}(1 - \cos\theta)$$

$$P_\gamma = hf = \frac{hc}{\lambda} \qquad P_\gamma' = \frac{hc}{\lambda'}$$

$$\boxed{(\lambda' - \lambda) = \frac{hc}{m}(1 - \cos\theta)}$$

from point of view of Q.M., photons are hardly to understand.

<u>Quantum Electrodynamics</u> gives
Klein-Nishina formula for scattering cross section

$$\frac{d\sigma}{d\Omega} = \frac{r_e^2}{2}\left[1 + a(1-c)\right]^{-2}\left[\underbrace{1 + c^2}_{\text{classical}} + \frac{a^2(1-c)^2}{1 + a(1-c)}\right]$$

$r_e = \frac{\hbar c \alpha}{mc^2} = 2.8$ fm $\qquad \alpha \cong \frac{1}{137}$ fine structure

$a = \frac{hf}{mc^2}$ $\qquad$ deviate from classical at large hf

de Broglie (1925) and Davisson, Germer (1928)

De Broglie hypothesized a matter wavelength according to, $\lambda = h/p$, with h Planck's constant. This formula is true relativistically!

Davisson and Germer measured the diffraction of electrons from nickel crystal. The also measured the diffraction with γ-rays and found the spacing $d = 0.091$ nm.

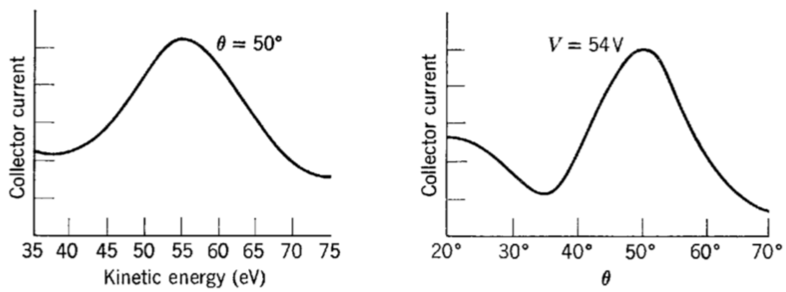


Figure 3-2 *Left:* The collector current in detector *D* of Figure 3-1 as a function of the kinetic energy of the incident electrons, showing a diffraction maximum. The angle θ in Figure 3-1 is adjusted to 50°. If an appreciably smaller or larger value is used, the diffraction maximum disappears. *Right:* The current as a function of detector angle for the fixed value of electron kinetic energy 54 eV.

Figure 1: Data of Davisson and Germer for electrons with $E_k = 54$ eV. From Eisberg and Resnick

The geometry gives the difference in path length Δ,

$$\Delta = 2\ell = 2d\cos\left(\frac{\theta}{2}\right)$$

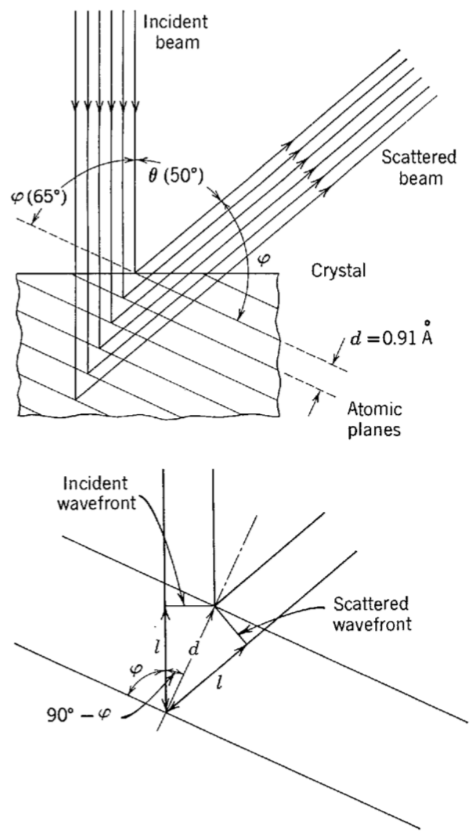


Figure 2: Illustration of diffraction geometry. From Eisberg and Resnick

$$E_k = \frac{p^2}{2m} = \left(\frac{h}{\lambda}\right)^2$$

At an accelerating potential of 1 Volt,

$$\lambda = \frac{1240\ \text{eV}\cdot\text{nm}}{\sqrt{2(1\ \text{eV})(5\times 10^5\ \text{eV})}} = 1.23\ \text{nm}$$

Theory predicts $\lambda \propto V^{-1/2}$ with slope 1.23 nm.

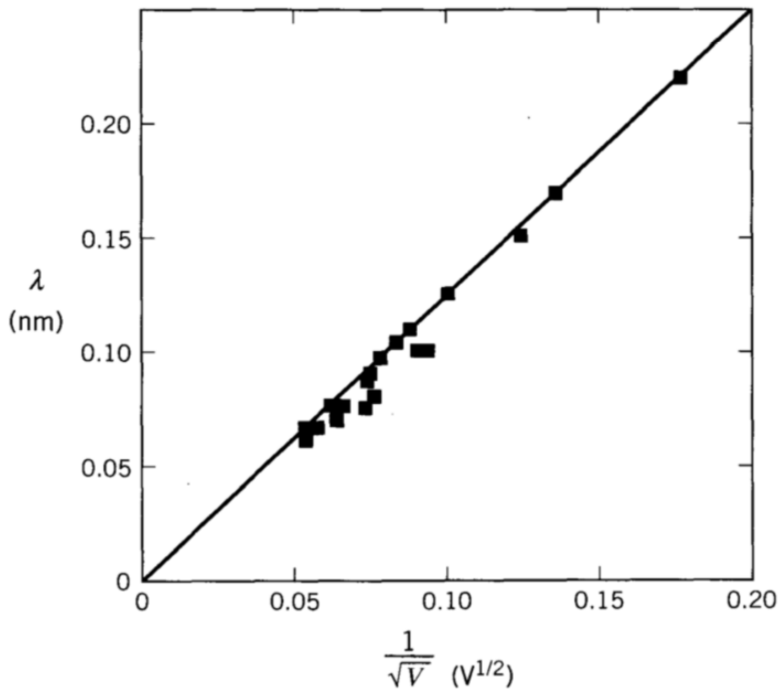


FIGURE 5-6 Testing the deBroglie equation for nonrelativistic electrons.
The electron wavelength is measured by measuring the scattering angle where the intensity maxima occur, using the Bragg condition, $n\lambda = 2d\sin\theta$. The wavelength is plotted as a function of the inverse square root of the accelerating voltage. The deBroglie equation predicts a linear relationship with a slope of 1.23 nm·$V^{1/2}$ (solid line). From C. J. Davisson, "Are Electrons Waves?," *Franklin Institute Journal* **205**, 597 (1928).

Figure 3: Fit of λ versus $V^{-1/2}$ has slope 1.23 nm.

So light is both wave and particle, and now an electron is both wave and particle. This known as wave-particle duality.