The Solar Neutrino Problem

effective solar fusion reaction

\[ 4p \rightarrow ^4\text{He} + 2e^+ + 2\nu_e + 25 \text{ MeV} \]
The predicted solar neutrino energy spectrum. The figure shows the energy spectrum of solar neutrinos predicted by the BP04 solar model. For continuum sources, the neutrino fluxes are given in number of neutrinos cm\(^{-2}\) s\(^{-1}\) MeV\(^{-1}\) at the Earth's surface. For line sources, the units are number of neutrinos cm\(^{-2}\) s\(^{-1}\). Total theoretical uncertainties are shown for each source. To avoid complication in the figure, we have omitted the difficult-to-detect CNO neutrino fluxes.
Raymond Davis of Brookhaven National Laboratory constructed a neutrino detector 1.6 km underground in the Homestake Gold Mine in Lead, South Dakota. The detector consists of a 378,000 liter tank of perchloroethylene, which is further isolated by being submerged in water. Theoretical expectations were about one neutrino-chlorine interaction per day, but the measured solar neutrino events were about a third of that, raising serious questions about the abundance of solar neutrinos.
Sudbury Neutrino Observatory, a 12-meter sphere filled with heavy water surrounded by light detectors located two thousand meters below the ground in Sudbury, Ontario, Canada.

\[ \nu_e \rightarrow W^+ + e^- \]
\[ \nu_\mu \rightarrow W^+ + \mu^- \]
\[ \nu_\tau \rightarrow W^+ + \tau^- \]
\[ \nu \rightarrow Z + \nu \]

\[ d + \nu \rightarrow p + n + \nu \]
Neutrino Mixing implies neutrino mass
Neutrino Mixing

For simplicity, 2 massive neutrino states masses \( m_1, m_2 \).

\[
\begin{pmatrix}
  \chi_1 \\
  \chi_2
\end{pmatrix}
= 
\begin{pmatrix}
  \cos \theta & \sin \theta \\
  -\sin \theta & \cos \theta
\end{pmatrix}
\begin{pmatrix}
  \chi_e \\
  \chi_\mu
\end{pmatrix}
\]

orthogonal \( \chi_1^* \chi_1 = \delta_{11} \)

at \( t = 0 \), produce \( \nu_e \) flavor

\[
\Psi_e(0) = \cos \theta \chi_1 + \sin \theta \chi_2
\]

Propagate as

\[
iE + \frac{1}{m_i} \chi_e
\]

ultra-relativistic

\[
E_i = \sqrt{p^2 + m_i^2} \approx p + \frac{m_i^2}{2p}
\]

Then there is a phase difference

\[
\Psi_e(t) = e^{-i\sqrt{m_2^2 - m_1^2}t} \left[ \cos \theta \chi_1 + e^{-i\delta_{21}} \sin \theta \chi_2 \right]
\]

\[
\delta_{21} = \frac{m_2^2 - m_1^2}{2pt}
\]

Probability to detect \( \nu_\mu \) flavor

\[
P(e \rightarrow \mu) = \left| \chi_\mu \Psi_e(t) \right|^2
\]
neutrino mixing:

\[ \nu_{\mu}^+ = -\sin \theta \nu_1^+ + \cos \theta \nu_2 \]

using orthogonality,

\[ \nu_{\mu}^+ \psi_e(t) = -\sin \theta \cos \phi + \sin \cos \phi e^{-i\Delta \phi} \]

\[ = -\sin \theta \cos \phi e^{-i\Delta \phi} \left[ e^{\frac{i\Delta m^2_{\mu e}}{2}} - e^{i\Delta \phi} \right] \]

\[ \Delta m^2 = \frac{\Delta m^2_{\mu e}}{2} \]

\[ P(e \rightarrow \mu) = |\nu_{\mu}^+ \psi_e(t)|^2 \]

\[ = \sin^2 \frac{\theta}{2} \sin^2 \left( \frac{\Delta \phi}{2} \right) \]

with \[ L = C, \quad CP = E \]

\[ P(e \rightarrow \mu) = \sin^2 \left( \frac{\theta}{2} \right) \sin \left( \frac{\Delta m^2_{\mu e}}{4E \cdot m_e} \right) \]

\[ \Delta m^2 = m_2^2 - m_1^2 \]

depends on square mass difference