April 20, 2021

Modern Physics: Test # 2

Pandemic rules:

- no books, notes, or computers
- two hours
- wash your hands

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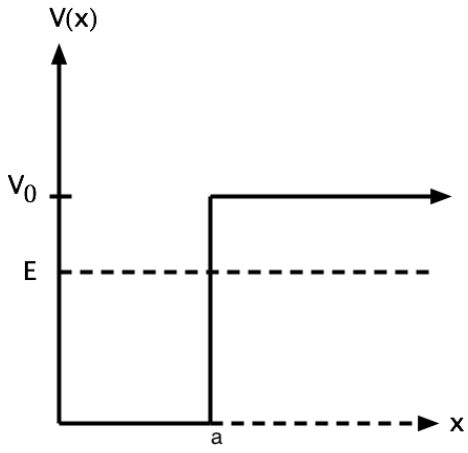
1. Consider QM particle in 1D bound state, half infinite potential (see figure).

   (a) As carefully as you can, sketch the ground state wave function on the figure.

   (b) Find the transcendental equation determining the energy

   (c) Find the condition on a in terms of particle mass and V_0 for a ground state to exist.



2. 1D QM harmonic oscillator: particle with mass m in potential

$$V(x) = \frac{1}{2} m\omega^2 x^2$$

   (a) Draw a sketch of the potential indicating the classical turning points for the ground state. What is value of the classical turning point for the ground state?

(b) What is the value of the uncertainty product $\Delta x \Delta p$ for the ground state? (This is something I expect you to just know, not to have to calculate. So if you don't know, it is best to just go on.)

(c) Use the uncertainty principle to estimate the ground state energy.

3. Hydrogen atom ground state

$$\psi(r,\theta,\phi)_{100} = \frac{1}{\sqrt{\pi} a_o^{3/2}} e^{-r/a_0}$$

(a) By explicit computation, find the most probable radius for the electron.

(b) Use the radial equation to find a_0 and the ground state energy in terms of m,α,$\hbar$, and c. (no points for remembering a_0 and E)

$$\frac{-\hbar^2}{2m}\left(\frac{d^2}{dr^2} + \frac{2}{r}\frac{d}{dr}\right) R + \frac{\hbar^2 \ell(\ell+1)}{2mr^2} R - \frac{\alpha \hbar c}{r} R = ER$$