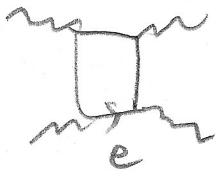
Spring 2021

Physics 450 HW#2 Solutions

2.2

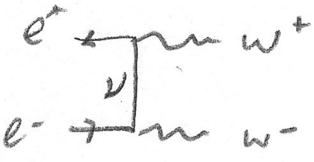


2.5

$\Xi^+ \to \begin{cases} \Lambda \pi^+ & \Delta S = 1 \text{ favored} \\ n \pi^+ & \Delta S = 2 \end{cases}$

2.6

$e^+ e^- \to W^+ W^-$



3 digrams

2.7

a ~~Q~~ charge

b EM

c ~~energy~~

d weak

e EM

f ~~L~~ (lepton #)

g strong

h weak

i strong

j strong

k ~~L~~, ~~B~~ (lepton, baryon #)

l strong

m strong

n ~~Q~~

o weak

p ~~Q~~

q EM

r weak

s weak

t strong

u EM

v weak

3.21

~~3.18~~

$\pi^0 \to \mu^- \bar{\nu}_\mu$ decay at rest

$$d = v\tau_\mu = \frac{p}{m_\mu}\tau_\mu$$

$$\tilde{p}_\mu = (E_\mu, p\hat{x}) \qquad \vec{p}_\nu = (E_\nu, -p_\nu\hat{x})$$

$$\overset{\vec{p}_\nu}{\longleftarrow}\ \overset{\vec{p}}{\longrightarrow}\ \hat{x} \qquad p_\nu = p$$

$$(m_\pi, \vec{0}) = (E_\mu + E_\nu, \vec{0})$$

$$E_\mu = \sqrt{m_\mu^2 + E_\nu^2}$$

$$m_\pi = \sqrt{m_\mu^2 + E_\nu^2} + E_\nu$$

$$E_\nu = \frac{m_\mu^2 - m_\pi^2}{2m_\pi}$$

$$d = \frac{\tau_\mu}{2m_\pi m_\mu}(m_\mu^2 - m_\pi^2)$$

2.25

(drop over tilde to do less writing)

~~3.22~~ (a) $s+t+u = (p_A+p_B)^2 + (p_A-p_C)^2 + (p_A-p_D)^2$

$$p_A^2+p_B^2+2p_A\cdot p_B + p_A^2+p_C^2-2p_A\cdot p_C$$

$$+p_A^2+p_D^2-2p_A\cdot p_D$$

$$= 3m_A^2+m_B^2+m_C^2+m_D^2 - 2p_A\cdot(\underbrace{p_C+p_D-p_B}_{=p_A})$$

$$p_A+p_B = p_C+p_D \longrightarrow$$

4 momentum conservation

$$= m_A^2+m_B^2+m_C^2+m_D^2$$

(b) in cm frame

$$\tilde{p}_A+\tilde{p}_B = (E_A, p) + (E_B, -p) = (E_A+E_B, \vec{0})$$

$$s = (E_A+E_B)^2 = E_A^2+E_B^2+2E_AE_B$$

and

$$\left.\begin{aligned} E_A^2 &= p^2+m_A^2 \\ E_B^2 &= p^2+m_B^2 \end{aligned}\right\} \quad E_A^2-E_B^2 = m_A^2-m_B^2$$

add

$$2E_A^2+2E_AE_B = s+m_A^2-m_B^2$$

$$2E_A(\underbrace{E_A+E_B}_{\sqrt{s}}) = s+m_A^2-m_B^2$$

$$E_A = \frac{1}{2\sqrt{s}}\left(s+m_A^2-m_B^2\right)$$

(c) In Lab frame

$$(E_a, \vec{p}) + (m_B, 0) = \tilde{P}_A + \tilde{P}_B$$

$$(E_A + m_B)^2 - p^2 = s$$

and $p^2 = E_A^2 - m_A^2$

$$2E_A m_B + m_B^2 + m_A^2 = s$$

$$E_A = \frac{1}{2m_B}(s - m_A^2 - m_B^2)$$

(d) In CM frame

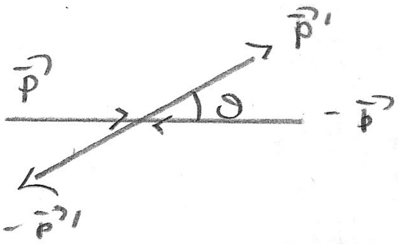
$$(E_a, \vec{p}) + (E_b, -\vec{p}) = \tilde{P}_A + \tilde{P}_B$$

$$(E_A + E_B)^2 = s$$

$$E_A + E_B = \sqrt{s}$$

3.26

~~3.23~~ $A + A \to A + A$ elastic scattering in cm frame



$|\vec{p}| = |\vec{p}\,'|$

$\vec{p} \cdot \vec{p}\,' = p^2 \cos\vartheta$

$\tilde{p}_A = (E, \vec{p})$ $\tilde{p}_C = (E, \vec{p}\,')$

$p_B = (E, -\vec{p})$ $\tilde{p}_D = (E, -\vec{p}\,')$

$$s = (\tilde{p}_A + \tilde{p}_B)^2 = (2E)^2 = 4(p^2 + m^2)$$

$$t = (\tilde{p}_A - \tilde{p}_C)^2 = (0, \vec{p} - \vec{p}\,') \cdot (0, \vec{p} - \vec{p}\,')$$

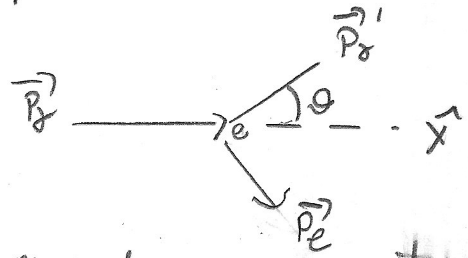
$$= -(\vec{p} - \vec{p}\,')^2 = -2p^2(1 - \cos\vartheta)$$

$$u = (\tilde{p}_A - \tilde{p}_D)^2 = (0, \vec{p} + \vec{p}\,') \cdot (0, \vec{p} + \vec{p}\,')$$

$$= -2p^2(1 + \cos\vartheta)$$

3.27

~~3.24~~ Compton



$E_\gamma = |\vec{P_\gamma}|$

m = e mass

4 momentum conservation

$$(E_\gamma, \vec{P_\gamma}) + (m, \vec{0}) = (E_\gamma', \vec{P_\gamma}') + (E_e, \vec{P_e})$$

$$(E_\gamma - E_\gamma', \vec{P_\gamma} - \vec{P_\gamma}') = (E_e - m, \vec{P_e})$$

$$E_\gamma - E_\gamma' = E_e - m$$

$$\vec{P_\gamma} - \vec{P_\gamma}' = \vec{P_e}$$

square

$$E_\gamma^2 + E_\gamma'^2 - 2E_\gamma E_\gamma' \cos\theta = P_e^2 = E_e^2 - m^2$$

$$E_\gamma^2 + E_\gamma'^2 - 2E_\gamma E_\gamma' = E_e^2 + m^2 - 2mE_e$$

$$2E_\gamma E_\gamma'(1 - \cos\theta) = -2m^2 + 2mE_e$$

$$E_\gamma E_\gamma'(1 - \cos\theta) = -m^2 + m(E_\gamma - E_\gamma' + m)$$

$$E_\gamma E_\gamma'(1 - \cos\theta) = m(E_\gamma - E_\gamma')$$

$$\frac{1}{m}(1 - \cos\theta) = \frac{1}{E_\gamma'} - \frac{1}{E_\gamma}$$

$$\frac{1}{E_\gamma'} = \frac{1}{E_\gamma} + \frac{1}{m}(1 - \cos\theta) \qquad E = \frac{h}{\lambda}$$

$$\lambda' = \lambda + \frac{h}{m}(1 - \cos\theta)$$

$\frac{h}{m}$ has dim. of [length], so put back the "c" as

$$\frac{h}{m} \to \frac{hc}{mc^2} \quad \left[\frac{\text{energy} \cdot \text{length}}{\text{energy}}\right] = [\text{length}]$$