Phys 450

Spring 2021

Hw #4 Solutions

4.26 By inspection of table (p. 47)

a) $Q = I_3 + \frac{1}{2}(A + S + B + C + T)$

b) $I_3 = \frac{1}{2}(U + D)$

$A = \frac{1}{3}\left[U + C + T - D - S - B\right]$

c) $Q = \frac{1}{3}\left[2(U + B + T) + (D + S + B)\right]$

4.27 $\vec{I} = \vec{I}_1 + \vec{I}_2 \qquad I^2 = I_1^2 + I_2^2 + 2\vec{I}_1 \cdot \vec{I}_2$

$\vec{I}_1 \cdot \vec{I}_2 = \frac{1}{2}\left(I^2 - I_1^2 + I_2^2\right)$

$\frac{1}{2} \otimes \frac{1}{2} = 1 \oplus 0$ triplet $I = 1$ $|1, m\rangle$, $m = -1, 0, 1$

singlet $I = 0$ $|0, 0\rangle$

$\langle 0,0 | \vec{I}_1 \cdot \vec{I}_2 | 0,0\rangle = \frac{1}{2}\left[0 - 2\left(\frac{3}{4}\right)\right] = -\frac{3}{4}$

$\langle 1,m | \vec{I}_1 \cdot \vec{I}_2 | 1,m\rangle = \frac{1}{2}\left[2 - 2\left(\frac{3}{4}\right)\right] = +\frac{1}{4}$

4.29

a) $\pi^- p \rightarrow K^0 + \Sigma^0$

b) $\pi^- p \rightarrow K^+ \Sigma^-$

c) $\pi^+ p \rightarrow K^+ \Sigma^+$

strange quark production requires CM energy

$$E_{CM} = m_K + m_\Sigma - m_\pi - m_p \simeq (500 + 1200 - 140 - 940)\text{ MeV}$$

$= 121$ MeV < 1232 MeV so allowed

Isospin conservation

Isospin of K is same as nucleon
Isospin of Σ is same as π

$|\pi^- p\rangle = |1,-1\rangle |\tfrac{1}{2}, \tfrac{1}{2}\rangle$

$|\pi^+ p\rangle = |1,1\rangle |\tfrac{1}{2} \tfrac{1}{2}\rangle$

$|\Sigma^0 K^0\rangle = |1,0\rangle |\tfrac{1}{2}, -\tfrac{1}{2}\rangle$

$|\Sigma^- K^-\rangle = |1,-1\rangle |\tfrac{1}{2}, \tfrac{1}{2}\rangle$

$|\Sigma^+ K^+\rangle = |1,1\rangle |\tfrac{1}{2}, \tfrac{1}{2}\rangle$

looking at C.G. coefficient p. 119

$A_a = \frac{\sqrt{2}}{3} m_{3/2} - \frac{\sqrt{2}}{3} m_{1/2}$

$A_b = \frac{1}{3} m_{3/2} + \frac{2}{3} m_{1/2}$

$A_c = m_{3/2}$

$\sigma_a = |A_a|^2$, etc.

if $I = 3/2$ dominates $\sigma_a : \sigma_b : \sigma_c = 2 : 1 : 9$

if $I = 1/2$ dominates $\sigma_a : \sigma_b : \sigma_c = 2 : 4 : 9$

4.31 Isospin conservation

$|\pi^+ p\rangle = |\tfrac{3}{2}, \tfrac{3}{2}\rangle$

$|\pi^- p\rangle = \frac{1}{\sqrt{3}} |\tfrac{3}{2}, -\tfrac{1}{2}\rangle - \sqrt{\tfrac{2}{3}} |\tfrac{1}{2}, -\tfrac{1}{2}\rangle$

looking at ratios of cross section $\pi^\pm p$ at resonances we see

$N(1525)$, $N(1688)$, $\Delta(1920)$, $N(2190)$

4.33

(a) isospin decomposition of 4 nucleons

$\tfrac{1}{2} \otimes \tfrac{1}{2} \otimes \tfrac{1}{2} \otimes \tfrac{1}{2} = (1 \oplus 0) \otimes (1 \oplus 0)$

$= 1 \otimes 1 \oplus 1 \oplus 1 \oplus 0$

$= 2 \oplus 1 \oplus 0 \oplus 1 \oplus 1 \oplus 0$

Check $2^4 = 16 = 5 + 3 + 1 + 3 + 3 + 1$

^4H (pnnn) is combination of $|2,-1\rangle$ and $|1,-1\rangle$

^4Li (pppn) is orthogonal combination of $|2,+1\rangle$ and $|1,+1\rangle$

Since neither exist, α is isospin singlet $(I = 0)$

(b) $d + d \to \alpha + \pi^0$

d (np) is isospin singlet, so reaction does not conserve isospin

(c) ^4Be (pppp) isospin $|2,+2\rangle$ cannot exist

nnnn isospin $|2,-2\rangle$ cannot exist

4.35)

(a) ν is Left handed not eigenstate of $\tilde{P}$

(b) The meson octet all have $J^{PC} = 0^{-+}$ odd parity

θ^+ decay is parity violating.

4.38) for A and B to mix they must have same mass, charge, B and L. Possible quark mixings are

$Q = -\frac{1}{3}$ $d\bar{s} \leftrightarrow \bar{s}d$, $d\bar{b} \leftrightarrow b\bar{d}$, $s\bar{b} \leftrightarrow \bar{b}s$

$Q = +\frac{2}{3}$ $u\bar{c} \leftrightarrow c\bar{u}$

We can have mixing of

$K^0 \leftrightarrow \bar{K}^0$, $B^0 \leftrightarrow \bar{B}^0$, $D^0 - \bar{D}^0$