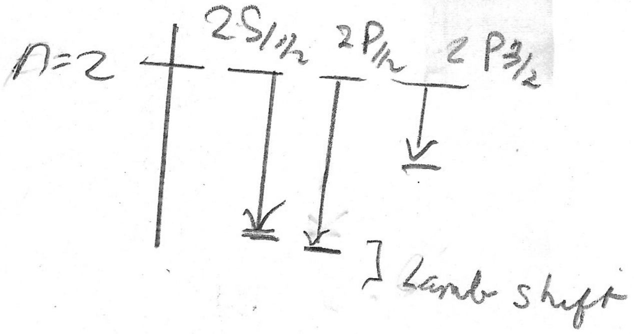
Phys 450 Spring 2021

HW #5 Solution

5.7 mult including e, p spin $= 4 \times n^2 \big|_{n=2} = 16$

distinct E levels ignoring hyperfine



3 distinct E levels

each is split due to hyperfine ⇒ 6 E levels

$$\Delta E_{hf}^{n=2} = \underbrace{\left(\frac{m}{m_p}\right)\alpha^4 mc^2 \, \frac{g_p}{16}}\;\frac{\pm 1}{(j \pm \frac{1}{2} + \frac{1}{2})(\ell + \frac{1}{2})}$$

$$\alpha = 1.37 \times 10^{-7}$$

Compare lower $2S_{1/2}$ to $2P_{1/2}$

$$\Delta E\,(s-p) = -\alpha\left[\frac{1}{\frac{1}{2}\cdot\frac{1}{2}} - \frac{1}{\frac{1}{2}\cdot\frac{3}{2}}\right] = -\alpha\left(\frac{8}{3}\right)$$

$$= -3.67 \times 10^{-7}$$

about (1/10) Lamb shift

HW#5-2

5.14 Construct 8 states ψ_{12} (antisymmetric 1,2 flavors)

3 flavors: $3 \otimes 3 \otimes 3 = 10 \oplus 8_{12} \oplus 8_{23} \oplus 1$

$I = 1/2, S = 0$

$$|n\rangle = \left(\frac{ud - du}{\sqrt{2}}\right) d \qquad |p\rangle = \frac{ud - du}{\sqrt{2}} u$$

$I = \frac{1}{2}, S = 2$

$$|\Xi^-\rangle = \frac{ds - sd}{\sqrt{2}} s \qquad |\Xi^+\rangle = \frac{us - su}{\sqrt{2}} s$$

$I = 1, S = 1$

$$|\Sigma^-\rangle = \left(\frac{ds - sd}{\sqrt{2}}\right) d \qquad |\Sigma^+\rangle = \left(\frac{us - su}{\sqrt{2}}\right) u$$

to get $I = 0$, apply isospin lowering operator

$$\hat{I}^- |i, m\rangle = \sqrt{i(i+1) - m(m-1)}\ |i, m\rangle$$

$$\hat{I}^- |1, 1\rangle = \sqrt{2}\ |1, 0\rangle = \sqrt{2}\ |\Sigma^0\rangle$$

$$= \left(\frac{ds - sd}{\sqrt{2}}\right) u + \frac{us - su}{\sqrt{2}} d$$

$$|\Sigma^0\rangle = \frac{1}{2}\left[(ds - sd) u + (us - su) d\right]$$

The remaining state $|\Lambda\rangle$ is linear combinations

$$|\Lambda\rangle = a(ds - sd)u + b(us - su)d + c(ud - du)s$$

$$\langle \Sigma^0|\Lambda\rangle = 0 \Rightarrow b = -a$$

$$\langle \psi_A|\Lambda\rangle = 0 \Rightarrow \quad |\psi_A\rangle = (uds - usd + dsu - dus + sud - sdu)\frac{1}{\sqrt{6}}$$

$$\langle \psi_A|\Lambda\rangle = a - b + c = 0 \qquad c = -2a$$

Normalizing

$$|\Lambda\rangle = a\left[(ds - sd)u - (us - su)d - 2(ud - du)s\right]$$

$$\langle\Lambda|\Lambda\rangle = 1 = a^2(2 + 2 + 4\cdot 2) = a^2\cdot 12$$

$$a = \frac{1}{\sqrt{12}}$$

5.18

Completely anti-symmetric in spin ψ and flavor ϕ

$$|\psi\rangle = A\left[\psi_{12}(\phi_{31}+\phi_{32})+\psi_{23}(\phi_{12}+\phi_{13}) + \psi_{31}(\phi_{23}+\phi_{21})\right]$$

proton $p = uud$

$$|p\uparrow\rangle = \frac{A}{2}\left[(\uparrow\downarrow\uparrow-\downarrow\uparrow\uparrow)(duu-uud+udu-uud) + (\uparrow\uparrow\downarrow-\uparrow\downarrow\uparrow)(udu-duu+uud-duu) + (\downarrow\uparrow\uparrow-\uparrow\uparrow\downarrow)(uud-udu+duu-udu)\right]$$

$$= \frac{A}{2}\left[\uparrow\downarrow\uparrow duu - \downarrow\uparrow\uparrow duu - \uparrow\uparrow\downarrow 2duu + \uparrow\downarrow\uparrow 2duu + \downarrow\uparrow\uparrow duu - \uparrow\uparrow\downarrow duu + \text{perm}\right]$$

$$= \frac{A}{2}\left[3\, d\uparrow u\downarrow u\uparrow - 3\, d\uparrow u\uparrow u\downarrow + \text{perm}\right]$$

5.20

p magnetic moment if spin-flavor antisymmetric

$$\mu \propto \mu_d \frac{\hbar}{2} + \mu_u\left(-\frac{\hbar}{2}\right) + \mu_u\left(+\frac{\hbar}{2}\right) = \frac{\hbar}{2}\mu_d < 0.$$

5.22 mass of Ξ (uss) spin $\frac{1}{2}$

$m_1 = m_2 = s \quad m_3 = m_u$

$$M = 2m_s + m_u + A'\left[\frac{\langle \vec{S}_1 \cdot \vec{S}_2 \rangle}{m_s^2} + \frac{\langle (\vec{S}_1 + \vec{S}_2) \cdot \vec{S}_u \rangle}{m_s m_u}\right]$$

$$\vec{J} = \vec{S}_1 + \vec{S}_2 + \vec{S}_u$$

$$J^2 = (\vec{S}_1 + \vec{S}_2)^2 + S_u^2 + 2(\vec{S}_1 + \vec{S}_2) \cdot \vec{S}_u$$

Ξ spin $1/2 \Rightarrow \langle J^2 \rangle = \hbar^2\, 3/4$

symmetry of ss flavor x spin $\Rightarrow$ spin $= 1$ for ss

$$\langle (\vec{S}_1 + \vec{S}_2) \rangle = 2\hbar^2$$

$$\langle (\vec{S}_1 + \vec{S}_2)^2 \rangle = 2\left(\tfrac{3}{4}\right)\hbar^2 + 2\vec{S}_1 \cdot \vec{S}_2 = 2\hbar^2$$

$$\langle \vec{S}_1 \cdot \vec{S}_2 \rangle = \tfrac{1}{4}\hbar^2$$

and $2\langle (\vec{S}_1 + \vec{S}_2) \cdot \vec{S}_u \rangle = \langle J^2 \rangle - \langle (\vec{S}_1 + \vec{S}_2)^2 \rangle - \langle S_u^2 \rangle$

$$= \left(\tfrac{3}{4} - 2 - \tfrac{3}{4}\right)\hbar^2 = -2\hbar^2$$

$$M = 2m_s + m_u + A'\hbar^2\left[\frac{1}{4m_s^2} - \frac{1}{m_s m_u}\right]$$

numerically $m_u = 363$ MeV, $m_s = 538$ MeV, $A'\hbar^2 = 100\, m_u$

gives $M = 1327$ MeV (1315 observed)