Phys 450 Spring 2021

Lec # 10

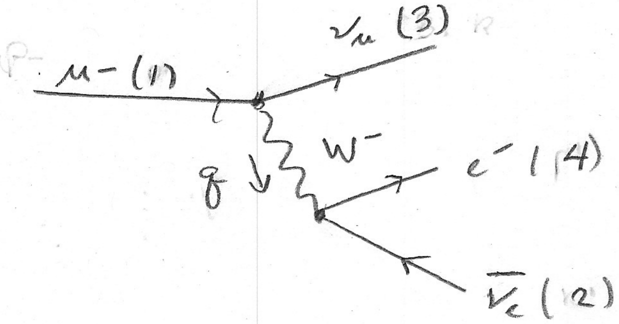
Weak Interactions

By experiment, interaction is V−A:

γ^μ vector $\gamma^\mu\gamma^5$ axial vector

Simplest process is muon decay (muon is Dirac point particle)

$$\mu^- \to e^- \bar{\nu}_e \nu_\mu \qquad \tau_\mu = 2.2\times10^{-6}\ \text{sec}$$

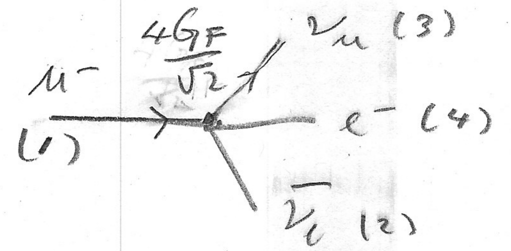


Vertex factor $\dfrac{-ig_w}{\sqrt{2}}\,\dfrac{1}{2}\gamma^\mu(1-\gamma^5)$ $\qquad g_w^2 = 4\pi\alpha_w \qquad \alpha_w = \dfrac{1}{29.5}$

Propagator $\dfrac{-i\left(g_{\mu\nu} - \dfrac{q_\mu q_\nu}{m_w^2}\right)}{q^2 - M_w^2}$ $\qquad M_w = 80.4\ \text{GeV}$

low q^2 Propagator $\longrightarrow \dfrac{i\,g_{\mu\nu}}{M_w^2}$

Effective low q^2 interaction is



$$\frac{4G_F}{\sqrt{2}} = \left(\frac{g_W}{\sqrt{2}}\right)^2 \frac{1}{m_W^2}$$

$G_F = 1.166 \times 10^{-5} / \mathrm{GeV}^2$

weak because m_W is heavy

$$\mathcal{M} = \frac{4G_F}{\sqrt{2}} J^\mu J_\mu^+$$

Leading order in perturbation theory
4-current interaction.

$$J^{e\mu} = \bar{V}_\nu \gamma^\mu (1-\gamma_5) V_e$$

$$J^{e+} = \bar{V}_e \gamma_\mu (1-\gamma_5) V_\nu$$

Muon decay $\mathcal{M} = \frac{g^2}{8 M_W^2} \bar{u}(3) \gamma^\mu (1-\gamma_5) u(1) \bar{u}(4) \gamma_\mu (1-\gamma_5) v_2$

$$\sum_{\text{Spin}} |\mathcal{M}|^2 = \left(\frac{g^2}{8 m_W^2}\right)^2 L^{e\mu\nu} L^\mu_{\mu\nu}$$

$$L^{e\mu\nu} = \mathrm{Tr}\left\{ \gamma^\mu (1-\gamma_5)(\not{p}_1 + m_e)(1-\gamma^5) \not{p}_3 \right\}$$

$$L^\mu_{\mu\nu} = \mathrm{Tr}\left\{ \gamma_\mu (1-\gamma_5) \not{p}_2 \gamma_\nu (\not{p}_4 + m_\mu) \right\}$$

See problem 9.2,

$$\sum_{spin} |m|^2 = 2\left(\frac{g_w}{m_w}\right)^4 (P_1 \cdot P_2)(P_3 \cdot P_4)$$

In muon rest frame $P_1 = (m, \vec{0})$ and neglect m_e

$$P_1 \cdot P_2 = m_\mu P_2^0 \qquad P_2 = (P_2^0, \vec{P_2}) \quad |\vec{P_2}| = P_2^0$$

$$P_3 \cdot P_4 = \frac{1}{2}(P_1 - P_2)^2 = \frac{1}{2}(m^2 - 2mP_2^0)$$

momentum conservation

$$\langle |m^2| \rangle = \frac{1}{2}\sum |m|^2 = \frac{1}{2}\left(\frac{g_w}{m_w}\right)^4 m^2 P_2^0 (m - 2P_2^0)$$

golden rule for 3 body decay

$$d\Gamma = \frac{\langle\,\rangle}{2m} (2\pi)^4 \delta^{(4)}(\,) \prod_{i=1}^{3} \frac{d^3 p_i}{(2\pi)^3 2E_i^0}$$

with $\delta^3(\vec{P_2} + \vec{P_3} + \vec{P_4})$ do P_3 (ν_μ) integral

$$d\Gamma = \frac{\langle\,\rangle}{2m} \left(\frac{1}{2\pi}\right)^5 \left(\frac{1}{2}\right)^3 \frac{d^3P_2\, d^3P_4}{P_2^0 P_4^0 E_3} \delta^0(m - P_2^0 - P_4^0 - E_3)$$

$$E_3 = |\vec{P_2} + \vec{P_4}|$$

next, do $d^3 P_2$ integral. E_3 depends on angle between $\vec{P_2}, \vec{P_4}$

For d^3P_2, choose polar axis $\vec{P_2}$. then

$$E_3^2 = (P_2^0)^2 + (P_4^0)^2 + 2\underbrace{\vec{P_2}\cdot\vec{P_4}}_{P_2^0 P_4^0 \cos\theta}$$

$$E_3\, dE_3 = P_2^0 P_4^0\, d(\cos\theta)$$

drop "0" superscript as no longer needed

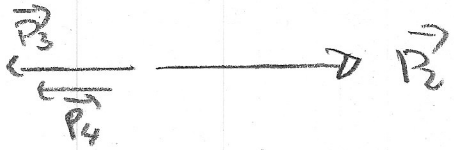
$$\frac{d^3P_2}{E_3} = (2\pi)\, P_2^2\, dP_2\, d(\cos\theta) = 2\pi\, \frac{P_2}{P_4} \cdot dE_3$$

use $\delta^0(m - P_2^0 - P_4^0 - E_3)$ for E_3 integral

Kinematic limits

$$E_3^\pm = \sqrt{P_2^2 + P_4^2 \pm P_2 P_4} = |P_2 \pm P_4|$$

$$E_3^- < m - P_2 - P_4 < E_3^+$$



$\max |\vec{P_2}| = \frac{1}{2} m$

$\cos\theta = -1$

$$d\Gamma = \frac{\langle\,\rangle}{m\, 2^4} \left(\frac{1}{2\pi}\right)^4 \frac{dP_2}{P_4^2}\, d^3P_4$$

with $\langle\,\rangle = m^2 P_2 (m - 2P_2)$

then limits on P_2 integral are from

$\frac{m}{2} - P_4$ to $\frac{m}{2}$

$$\frac{d\Gamma}{d^3P_4} = \left(\frac{g}{4\pi m_W}\right)^4 \frac{m}{P_4^2} \int_{\frac{m}{2}-P_4}^{m/2} P_2(m-2P_2)\,dP_2$$

$$= \left(\frac{g}{4\pi m_W}\right)^4 m \left(\frac{m}{2} - \frac{2}{3}P_4\right)$$

Energy spectrum of electron (P_4)

$$\frac{d\Gamma}{dP_4} = \left(\frac{g}{m_W}\right)^4 \frac{m^2 P_4}{2(4\pi)^3}\left(1 - \frac{4P_4}{3m}\right)$$ see fig 9.1

and muon lifetime:

$$\tau_\mu = \frac{1}{\Gamma} = \left(\frac{m_W}{m_\mu g}\right)^4 \frac{12(8\pi)^3}{m_\mu} = \frac{192\pi^3}{G_F^2 m_\mu^5}$$

given $G_F = 1.166 \times 10^{-5}$

Structure of weak Interactions

chiral Dirac spinor wave functions (states)

$$u_L(p) = \tfrac{1}{2}(1-\gamma^5)\,u(p) \qquad u_R(p) = \tfrac{1}{2}(1+\gamma^5)\,u(p)$$

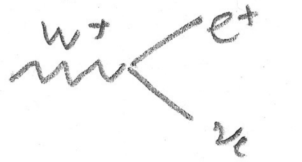
$$v_L(p) = \tfrac{1}{2}(1+\gamma^5)\,v(p) \qquad v_R(p) = \tfrac{1}{2}(1-\gamma^5)\,v(p)$$

Chiral wave functions are Lorentz invariant.
Helicity is not L.I. because boost changes sign of $\vec{p}$ but not $\vec{S} = \frac{1}{2}\vec{\sigma}$ eigenvalue. In high energy limit ($m \approx 0$) they are the same.

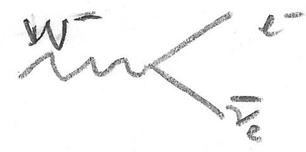
Then weak current *

In QFT, destroys e^-, create e^+

$$j^+_\mu = \bar{\nu}_e \gamma_\mu \left(\frac{1-\gamma^5}{2}\right) e = \bar{\nu}_{eL} \gamma_\mu e_L$$



$$\left(j^+_\mu\right)^\dagger = j^-_\mu = \bar{e}\,\gamma_\mu \left(\frac{1-\gamma^5}{2}\right) \nu_e = \bar{e}_L \gamma_\mu \nu_{eL}$$



$W^\pm$ couple only to left-handed chiral currents.

$$\mathcal{L}_W = \frac{-ig}{\sqrt{2}}\left(j^+_\mu W^{\mu +} + j^-_\mu W^{\mu -}\right) \quad \text{interaction term}$$

$W^{\mu +}$ – destroys W^+, creates W^-

$W^{\mu -}$ – destroys W^-, creates W^+

*error in Griffiths

Weak isospin

Analogous to spin, introduce weak isospin symmetry

use $\vec{\tau}$ for Pauli matrices for this internal space.

doublet $\quad \chi_L = \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L$

$I_W = \frac{1}{2}$

$\tau_+ = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$ raising operator

$\tau_\pm = \frac{1}{2}(\tau_1 \pm i\tau_2)$

$\tau_- = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$ lowering operator

Then weak currents can be written as

$$j_\mu^+ = \bar{\nu}_L \gamma_\mu e_L = (\bar{\nu}_e, \bar{e}) \gamma_\mu \tau_+ \begin{pmatrix} \nu_e \\ e \end{pmatrix} = \chi_L^\dagger \gamma_\mu \tau_+ \chi_L$$

$$j_\mu^- = \chi_L^\dagger \gamma_\mu \tau_- \chi_L$$

Note that the electromagnetic current

(includes charge in units of e. see Gell-Mann Nishijimma)

$$j^\mu_{EM} = -\bar{e}\gamma^\mu e = -(\bar{e}_L + \bar{e}_R)\gamma^\mu(e_L + e_R) = -\bar{e}_L\gamma^\mu e_L - \bar{e}_R\gamma^\mu e_R$$

where cross terms don't appear –

$$\bar{e}_L\gamma^\mu e_R = \bar{e}\left(\frac{1+\gamma_5}{2}\right)\gamma_\mu\left(\frac{1+\gamma_5}{2}\right)e = \bar{e}\gamma^\mu\left(\frac{1-\gamma_5}{2}\right)\left(\frac{1+\gamma_5}{2}\right)e$$

$$= \bar{e}\gamma^\mu \frac{1}{4}(1-\gamma_5^2)\, e = 0$$

Photon couples equally to L, R chiral state.

Conserves parity

The W field is a Vector in weak isospin space, analogous to Pion : π^+, π^-, π^0

It is in adjoint representation of SU(2)

$2^2 - 1 = 3$ States

$\vec{W} \equiv (W_+, W_-, W_3)$ iso-vector field of spin-1 Bosons

W^3 couples to

$$j^{3\mu} = \bar{\chi}_L \gamma_\mu \left(\frac{\tau_3}{2}\right) \chi_L$$

$$= \frac{1}{2} (\bar{\nu}_L, \bar{e}) \gamma_\mu \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}$$

$$= \frac{1}{2} \left(\bar{\nu}_L \gamma_\mu \nu_L - \bar{e}_L \gamma_\mu e_L \right)$$

We can write interaction of $\vec{W}$ as

$$g \vec{j}_\mu \cdot \vec{W} = g \bar{\chi}_L \gamma_\mu \frac{\vec{\tau}}{2} \cdot \vec{W} \chi_L$$

$\vec{W}$ couples only to left handed chiral fields (particles)

Weak Hypercharge

$T_3 \equiv$ eigenvalue of $\frac{\tau^3}{2}$ weak isospin

Analogous to Gell-Mann Nishijima relation

$$Q = T_3 + \frac{1}{2} Y_W$$

Lepton	T_3	Q	Y_W
ν_e	$1/2$	0	-1
e_L^-	$-1/2$	-1	-1
e_R^-	0	-1	-2

Quark	T_3	Q	Y_W
u_L	$\frac{1}{2}$	$2/3$	$1/3$
d_L	$-\frac{1}{2}$	$-1/3$	$1/3$
u_R	0	$2/3$	$4/3$
d_R	0	$-1/3$	$-2/3$

Weak hypercharge current $Y_W = 2(Q - T_3)$

$$j^\mu_{Y_W} = 2\left(j^\mu_{EM} - j^3_\mu\right)$$

where j_μ_EM = -ēγ_μe
includes (L+R)

Introduce weak iso-scalar vector Boson B^μ
interaction and coupling g'

$$\frac{g'}{2} j^\mu_{Y_W} B_\mu$$

Full $\underset{g}{SU(2)}\ \underset{g'}{U(1)}$ interaction is

$$-i\left[g\,\underset{\uparrow}{\vec{J}^{\mu}}\cdot\vec{W}_{\mu} + \frac{g'}{2}\,\underset{\uparrow}{j^{\mu}_{Y}}\,B^{\mu}\right]$$

weak current   weak hypercharge current

Electroweak Theory (Ewk) Glashow, Weinberg, Salam

Mix W^3, B vector Bosons to get

Z, A (photon) $\theta_W \equiv$ Weinberg angle

$$A^{\mu} = B^{\mu}\cos\theta_W + W^{3\mu}\sin\theta_W$$

$$Z^{\mu} = -B^{\mu}\sin\theta_W + W^{3\mu}\cos\theta_W$$

Weinberg rotation

$$\begin{pmatrix} A^{\mu} \\ Z^{\mu} \end{pmatrix} = \begin{pmatrix} \cos\theta_W, & \sin\theta_W \\ -\sin\theta_W, & \cos\theta_W \end{pmatrix}\begin{pmatrix} B^{\mu} \\ W^{3\mu} \end{pmatrix}$$

$$g\,j^{3\mu}\,W^3_{\mu} + \frac{g'}{2}\,j^{\mu}_{Y}\,B_{\mu} =$$

$$\left(g\sin\theta_W\, j^{3\mu} + \frac{g'}{2}\cos\theta_W\, j^{\mu}_{Y}\right) A_{\mu}$$

$$+ \left(g\cos\theta_W\, j^{3\mu} - \frac{g'}{2}\sin\theta_W\, j^{\mu}_{Y}\right) Z_{\mu}$$

with $j^{\mu}_{Y} = 2(j^{\mu}_{EM} - j^{3\mu})$

A_μ term is

$g \sin\theta\, j^{3\mu} + g' \cos\theta\, (j^{\mu}_{EM} - j^{3\mu}) =$

$\underbrace{(g \sin\theta - g' \cos\theta)}_{=0} j^{3\mu} + \underbrace{g' \cos\theta}_{= g_e \text{ (electric charge)}}\, j^{\mu}_{EM} = \boxed{g_e\, j^{\mu}_{EM}}$

$g \sin\theta = g' \cos\theta$ & $g' \cos\theta = g_e$

Z_μ term is

$g \cos\theta_W\, j^{3\mu} - g' \sin\theta_W \left[j^{\mu}_{EM} - j^{3\mu} \right]$

$= (g \cos\theta_W + g' \sin\theta_W)\, j^{3\mu} - \underbrace{g' \sin\theta_W}_{g \sin^2\theta_W / \cos\theta_W}\, j^{\mu}_{EM}$

$\dfrac{g \cos^2\theta_W}{\cos\theta_W} + g \dfrac{\sin^2\theta_W}{\cos\theta_W} = \dfrac{g}{\cos\theta_W}$

$\dfrac{g}{\cos\theta_W} = \dfrac{g'}{\sin\theta_W} = \dfrac{g_e}{\cos\theta_W \sin\theta_W}$

Z_μ term is then

$$\boxed{\frac{g_e}{\sin\theta_W \cos\theta_W} \left[j^{3\mu} - \sin^2\theta_W\, j^{\mu}_{EM} \right]}$$

sin^2(θ_W) = 0.23

Weak and EM interactions unified

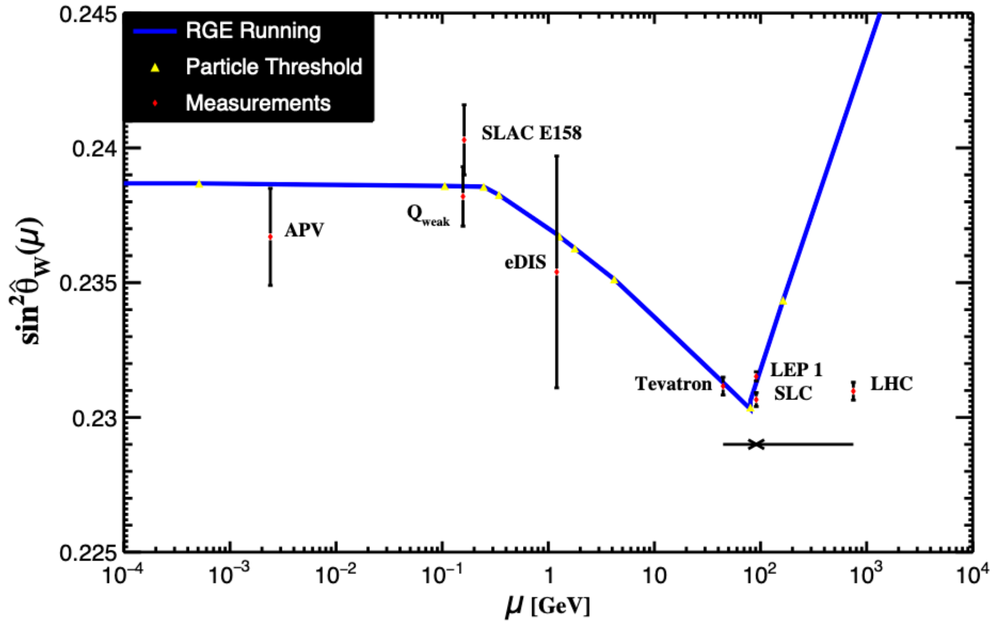


Figure 10.2: Scale dependence of the weak mixing angle defined in the $\overline{\text{MS}}$ scheme [35,74] (for the scale dependence in a mass-dependent renormalization scheme, see Ref. [73]). The minimum of the curve corresponds to $\mu = M_W$, below which we switch to an effective theory with the $W^\pm$ bosons integrated out, and where the β-function for $\widehat{s}^2(\mu)$ changes sign. At M_W and each fermion mass there are also discontinuities arising from scheme dependent matching terms, which are necessary to ensure that the various effective field theories within a given loop order describe the same physics. However, in the $\overline{\text{MS}}$ scheme these are very small numerically and barely visible in the figure provided one decouples quarks at $\mu = \widehat{m}_q(\widehat{m}_q)$. The width of the curve exceeds the theory uncertainty from strong interaction effects which at low energies is at the level of $\pm 2 \times 10^{-5}$ [35]. The Tevatron and LHC measurements are strongly dominated by invariant masses of the final-state di-lepton pair of $\mathcal{O}(M_Z)$ and can thus be considered as additional Z pole data points. For clarity we displayed the Tevatron and LHC points horizontally to the left and right, respectively.