Phys 450 Spring 2021

lec # 11

<u>Weak interactions of Quarks</u>

Three generations of weak isodoublet fermions:

leptons $\begin{pmatrix} \nu_e \\ e \end{pmatrix} \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix} \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}$

quarks $\begin{pmatrix} u \\ d' \end{pmatrix} \begin{pmatrix} c \\ s' \end{pmatrix} \begin{pmatrix} t \\ b' \end{pmatrix}$

Flavor changing weak interactions do not couple diagonally to quark mass (propagating) eigenstates. By convention, rotate bottom quark doublet.

CKM matrix V

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \left(\begin{array}{cc|c} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ \hline V_{td} & V_{ts} & V_{td} \end{array}\right) \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

weak interaction

mass, propagating

upper left 2x2 parameterized by Cabibbo angle θ_c

V is <u>unitary</u> $V^\dagger V = I$

V has 3 angles, 1 complex phase

#11-2

Weak Currents of Quarks

$$j^{+\mu} W^+_\mu = (\bar{u}, \bar{c}, \bar{t}\,)\,\gamma^\mu \left(\frac{1-\gamma_5}{2}\right) V \begin{pmatrix} d \\ s \\ b \end{pmatrix} W^+_\mu$$

destroys W^+
creates W^-

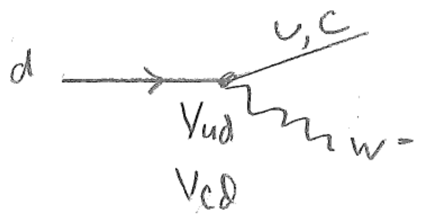
$$j^{-\mu} = (j^{+\mu})^\dagger$$

$$j^{-\mu} W^-_\mu = (\bar{d}, \bar{s}, \bar{b}\,) V^\dagger \gamma^\mu \left(\frac{1-\gamma_5}{2}\right) \begin{pmatrix} u \\ c \\ t \end{pmatrix} W^-_\mu$$

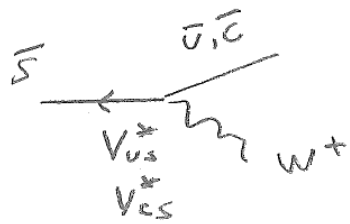
For 2 generations

$$\begin{pmatrix} d' \\ s' \end{pmatrix} = \begin{pmatrix} \cos\theta_c\,, & \sin\theta_c \\ -\sin\theta_c\,, & \cos\theta_c \end{pmatrix} \begin{pmatrix} d \\ s \end{pmatrix}$$

$j^{+\mu}$:



$j^{-\mu}$:



$V_{ud} = \cos\theta_c$ $\qquad$ $V^*_{us} = \sin\theta_c$

$V_{cd} = -\sin\theta_c$ $\qquad$ $V^*_{cs} = \cos\theta_c$

$$(V^*_{us}, V^*_{cs}) \begin{pmatrix} V_{ud} \\ V_{cd} \end{pmatrix} = 0 = \sum_i V^*_{2i} V_{i1} = (V^\dagger V)_{21}$$

Cancellation in GIM mechanism

\# 11-3

<u>GIM</u> Absence of flavor changing neutral currents (FCN). Because $V^\dagger V = I$

Third component of weak isospin current

$$j^{3\mu} = \sum_{\text{generation}} \chi_L \gamma_\mu \frac{\tau_3}{2} \chi_L = (\bar{u}\ \bar{d}')\gamma_\mu\left(\frac{1-\gamma_5}{2}\right)\frac{\tau_3}{2}\begin{pmatrix} u \\ d' \end{pmatrix}$$

2 for simplicity

$$+ (\bar{c}\ \bar{s}')\gamma_\mu\left(\frac{1-\gamma_5}{2}\right)\frac{\tau_3}{2}\begin{pmatrix} c \\ s' \end{pmatrix}$$

$$= \frac{1}{2}\left[\bar{u}_L \gamma^\mu u_L + \bar{c}_L \gamma_\mu c_L\right] \quad \text{up}$$

$$- \frac{1}{2}\left[\bar{d}_L \gamma_\mu d'_L + \bar{s}'_L \gamma_\mu s'_L\right] \quad \text{down}$$

but because $V^\dagger V = 1$,

$$\bar{d}'_L \gamma_\mu d'_L + \bar{s}'_L \gamma_\mu s'_L = \bar{d}_L \gamma_\mu d_L + \bar{s}_L \gamma_\mu s_L$$

So

$$j^{3\mu} = \sum_{\text{quarks}} T_3^i\, \bar{q}_i \gamma_\mu\left(\frac{1-\gamma_5}{2}\right) q_i$$

$$T_3^{up} = +\frac{1}{2}$$

$$T_3^{down} = -\frac{1}{2}$$

Z^{μ} coupling is complicated;

$$j^{Z\mu} = j^{3\mu} - \sin^2\theta_W \, j^{\mu}_{EM}$$

$$j^{\mu}_{EM} = \sum_{\text{quarks}} Q_i \, \bar{q}_i \gamma_\mu q_i$$

Q_i quark charge in units of e

$$j^{Z\mu} = \sum_{\text{quarks}} \bar{q}_i \gamma_\mu \left(\frac{c_V^i - c_A^i \gamma^5}{2} \right) q_i$$

$$c_V^i = T_3^i - \cancel{2}\sin^2\theta_W \, Q_i$$

$$C_A^i = T_3^i$$

$$C_V^{up} = \frac{1}{2} - \frac{4}{3}\sin^2\theta_W \cong 0.19$$

$$C_V^{down} = -\frac{1}{2} + \frac{2}{3}\sin^2\theta_W = -0.34$$

and for leptons

$$C_V^{\nu} = \frac{1}{2}$$

$$C_V^{e} = -\frac{1}{2} + 2\sin^2\theta_W = \underline{\underline{-0.03}}$$

$\sin^2\theta_W$ = 0.23 close to $\frac{1}{4}$

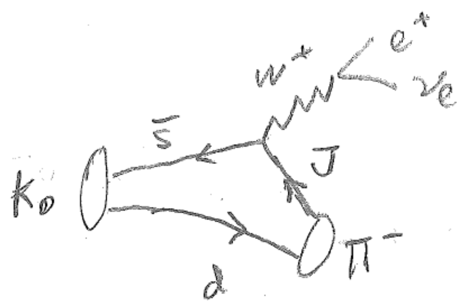
#12-5

GIM mechanism highly suppresses "neutral current" decays in perturbation theory

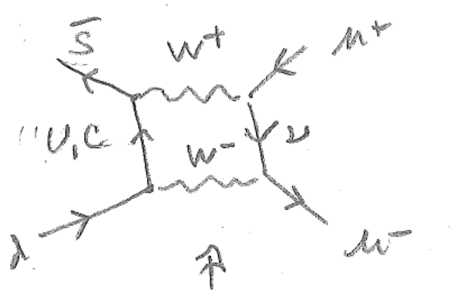
example neutral kaon leptonic decay

$Br(K^0 \to \pi^- e^+ \bar{\nu}_e) = 0.40$

$Br(K^0 \to \mu^+ \mu^-) = 6.8 \times 10^{-9}$



Charged current



"neutral current"

$\bar{s} \to \bar{c}\, W^+ \quad V_{cs}^* = \cos\theta_c$

$\bar{s} \to \bar{u}\, W^+ \quad V_{us}^* = \sin\theta_c$

$d \to c\, W^- \quad V_{cd} = -\sin\theta_c$

$d \to u\, W^- \quad V_{ud} = \cos\theta_c$

adding u, c amplitudes:

$M_u \propto \sin\theta_c \cos\theta_c$

$M_c \propto -\sin\theta_c \cos\theta_c$

Note. diagram in Griffiths P. 3.27 draws arbitrary $\mu^+ - \mu^-$ current line in direction that is very confusing!