Lecture #14 Higgs Mechanism

Gauge Group $SU_L(2) \times U_Y(1)$

$(g)\ W^+, W^0, W^- ; (g')\ B$

$$Q = I_3 + \frac{Y}{2}$$

mix $\rightarrow Z^0, A$

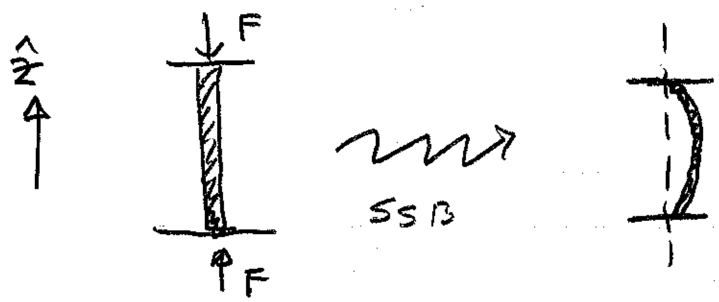
$$\frac{g'}{g} = \tan\theta_W$$

$$e = gg'/\sqrt{g^2+g'^2}$$

$$SU_L(2) \times U_Y(1) \xrightarrow[SSB]{} U_{EM}(1)$$

Spontaneous Symmetry Breaking (SSB)

Classical analog - compressed rod



Bowed rod

H is rotationally invariant about z-axis, but ground state is not symmetric; special angle ϕ

Rotations about ϕ require no energy - 1 massless Goldstone boson for each broken symmetry.

Radial oscillations have large energy (mass). Analog of the Higgs Boson.

Some simple examples (from Quigg, <u>Gauge Theories</u>)

<u>Example 1</u>: Discrete symmetry

$\phi(\bar{x})$ scalar field

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^2 - \frac{1}{2}\mu^2\phi^2$$

$\rightarrow$ eq. of motion, $\partial^2\phi + \mu^2\phi = 0$ Klein-Gordon

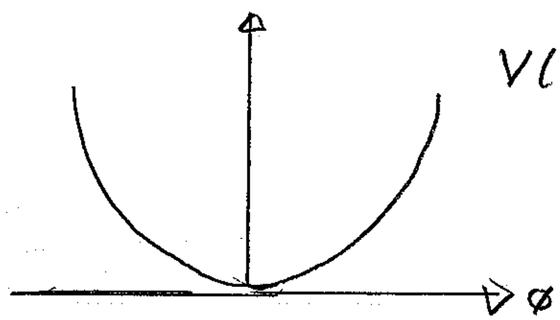
free, massive scalar field, $\partial^2 = \partial^\mu\partial_\mu = \left(\frac{\partial^2}{\partial t^2} - \nabla^2\right)$

Add self interaction:

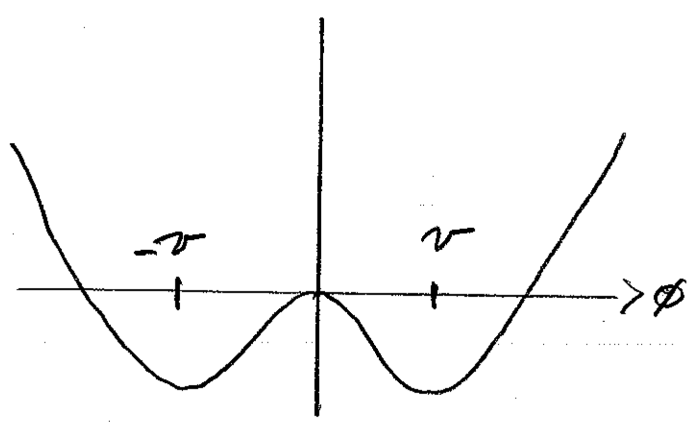
$$V_{self}(\phi) = \frac{1}{4}\lambda\phi^4$$

$\mathcal{L}$ is invariant under $\phi \rightarrow -\phi$

$$V(\phi) = \frac{1}{2}\mu^2\phi^2 + \frac{\lambda}{4}\phi^4$$



Suppose that $\mu^2 < 0$. Then $V(\phi)$ has a minimum



$$\left.\frac{dV}{d\phi}\right|_v = \mu^2\phi + \lambda\phi^3 = \phi\left[-(-\mu^2) + \lambda\phi^2\right] = 0$$

$$v = \pm\sqrt{-\mu^2/\lambda}$$

Ground state will be either $\langle\phi\rangle = \pm v$

breaking parity.

-μ^2 = λv

Oscillations about minimum, $\phi' \equiv \phi - v$

$$\mathcal{L} = \frac{1}{2}(\partial_\mu\phi')^2 + \frac{1}{2}\lambda v^2(\phi'+v)^2 - \frac{\lambda}{4}(\phi'+v)^4$$

Expanding and ignoring irrelevant constant terms:

$$\mathcal{L} = \frac{1}{2}(\partial_\mu\phi')^2 - \lambda v^2\phi'^2 - \lambda v\phi'^3 - \frac{\lambda}{4}\phi'^4$$

mass term with correct sign

<u>Example 2:</u> breaking of continuous symmetry and Goldstone Boson

$\phi = (\phi_1 + i\phi_2)/\sqrt{2}$ complex scalar field describes charged scalar particle.

$\phi^*\phi = \frac{1}{2}(\phi_1^2 + \phi_2^2)$

$$\mathcal{L} = \partial_\mu\phi^*\partial^\mu\phi - \mu^2\phi^*\phi - \lambda^2|\phi^*\phi|^2$$

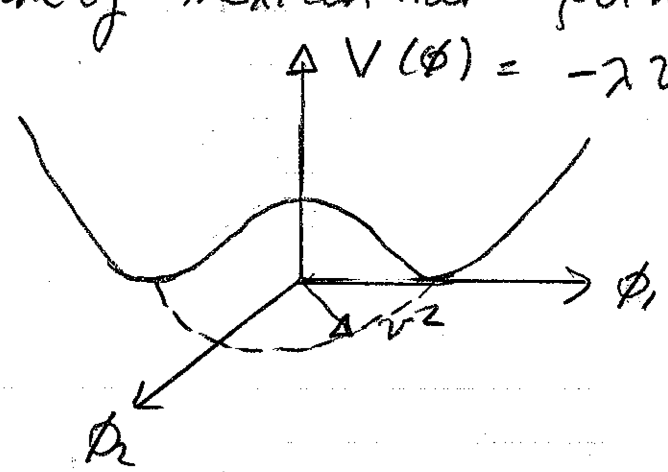
$\mathcal{L}$ is invariant under global (constant) phase symmetry $\phi \to e^{i\alpha}\phi$

For $\mu^2 = -\lambda v^2 < 0$, V has minimum for $v^2 = \phi_1^2 + \phi_2^2$.

real, so in 1 direction

Choose $\langle\phi\rangle = v/\sqrt{2}$ and expand about minimum of "Mexican Hat" potential:

$$V(\phi) = -\lambda v^2|\phi|^2 + \lambda^2|\phi|^4$$



$$\phi(x) = \frac{1}{\sqrt{2}}\left(v + \eta(x) + i\xi(x)\right)$$

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \eta)^2 + \frac{1}{2}(\partial_\mu \xi)^2 + \lambda^2 v^2\left[(v+\eta)^2 + \xi^2\right] - \lambda\left[(v+\eta)^2 + \xi^2\right]^2$$

$$= \frac{1}{2}(\partial_\mu \eta)^2 + \frac{1}{2}(\partial_\mu \xi)^2 - v^2\lambda\eta^2 + \text{cubic} + \text{quartic} + \text{const}$$

$\eta \rightarrow$ massive Boson broken direction

$\xi \rightarrow$ massless (Goldstone) Boson

STANDARD MODEL

Simplest choice is $Y_W = +1$ weak isodoublet $I = \frac{1}{2}$

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_1 + i\phi_2 \\ \phi_3 + i\phi_4 \end{pmatrix}$$

Two complex, four real, scalar fields

most general, renormalizable potential

$$V(\phi) = \mu^2(\phi^\dagger\phi) + \lambda(\phi^\dagger\phi)^2 \qquad \lambda \text{ real}$$

for $\mu^2 < 0$, minimum

$$\phi^\dagger\phi = \frac{-\mu^2}{\lambda} \equiv \left(\frac{v}{\sqrt{2}}\right)^2$$

choose broken direction $\langle vac | \phi_3 | vac \rangle = v$

$$\langle vac | \phi | vac \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}$$

Electric charge $\hat{Q} = \hat{I}^3 + \frac{\hat{Y}}{2}$

$$\langle Q \rangle_{vac} = -\frac{1}{2} + \frac{1}{2} = 0$$

Electric charge remains conserved

ϕ_1, ϕ_2, ϕ_4 are massless Goldstone Bosons

ϕ_3 massive Higgs

To make symmetry clearer

$$\phi(x) = e^{-i\vec{I}\cdot\vec{\theta}(x)} \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v + h(x)\end{pmatrix}$$

h is physical Higgs field

ϕ^+, ϕ^-, ϕ^3 become longitudinal polarization of <u>massive</u> W^+, W^-, Z^0

$\vec{I} = \left(\frac{\tau_+}{2}, \frac{\tau_-}{2}, \frac{\tau_3}{2}\right)$ weak isospin

Scalar portion of Lagrangian

$$\mathcal{L}_\phi = \left|\left(i\partial_\mu - g\vec{I}\cdot\vec{W}_\mu - g'\frac{Y}{2}B_\mu\right)\phi\right|^2 - V(\phi)$$

Coupling of ϕ is

$$\left|\left(g\vec{I}\cdot\vec{W}_\mu - \frac{g'}{2}B_\mu\right)\phi\right|^2 \quad \text{with } \phi = \frac{1}{\sqrt{2}}\begin{pmatrix}0\\v\end{pmatrix}$$

we find gauge Boson masses $\left(\vec{I} = \frac{1}{2}\vec{\tau}\right)$

[mass] =

$$\frac{1}{8}\left|\left(g\underbrace{\vec{\tau}\cdot\vec{W}_\mu} - g' B_\mu\right)\begin{pmatrix}0\\v\end{pmatrix}\right|^2$$

$$\begin{pmatrix} W^3_\mu, & W^1_\mu - iW^2_\mu \\ W^1_\mu + iW^2_\mu, & -W^3_\mu\end{pmatrix} = \begin{pmatrix} W^3_\mu, & \sqrt{2}W^-_\mu \\ \sqrt{2}W^+_\mu, & -W^3_\mu\end{pmatrix}$$

$$[\text{mass}] = \frac{1}{8}\left|\left(\begin{matrix} gW_\mu^3 + g'B_\mu\,, & \sqrt{2}\,g W_\mu^- \\ \sqrt{2}\,g W_\mu^+ & , -gW_\mu^3 + g'B_\mu \end{matrix}\right)\left(\begin{matrix} 0 \\ v \end{matrix}\right)\right|^2$$

$$= \frac{v^2}{8}\left(\sqrt{2}\,g W^{+\mu}, -gW^{3\mu} + g'B^\mu\right)\left(\begin{matrix} \sqrt{2}\,g W_\mu^- \\ -gW_\mu^3 + g'B_\mu \end{matrix}\right)$$

$$= \frac{v^2}{4} g^2 W^{+\mu} W_\mu^- + \frac{v^2}{8}\left|\left(gW_\mu^3 - g'B_\mu\right)\right|^2$$

with $g'/g \equiv \tan\theta_w$

$$gW_\mu^3 + g'B_\mu = \frac{g}{\cos\theta_w}\left(\cos\theta_w W_\mu^3 - \sin\theta_w B_\mu\right) = \frac{g}{\cos\theta_w} Z^\mu$$

orthogonal $\left(\sin\theta_w W_\mu^3 + \cos\theta_w B_\mu\right) = A_\mu$

does not appear. Photon is massless.

$$[\text{mass}] = \underbrace{\left(\frac{vg}{2}\right)^2}_{= m_W^2} W^{+\mu} W_\mu^- + \frac{1}{2}\underbrace{\left(\frac{vg}{2\cos\theta_w}\right)^2}_{= \frac{1}{2} M_Z^2} Z^\mu Z_\mu$$

$\frac{1}{2}$ for neutral vect. Vector Boson

$$m_W = \frac{vg}{2} \qquad \boxed{m_Z = \frac{m_W}{\cos\theta_w}}$$

$$G_F = \frac{g^2}{8m_W^2} = \frac{1}{2v^2} \Rightarrow v = 246\text{ GeV}$$

$$m_W = \frac{37.3\text{ GeV}}{\sin\theta_w}\,, \quad M_Z = \frac{74.6\text{ GeV}}{\sin 2\theta_w}$$

Higgs Mass

$$V(\phi) = \mu^2\phi^\dagger\phi + \lambda(\phi^\dagger\phi)^2 = -\frac{\lambda v^2}{2}(\phi^\dagger\phi) + \lambda(\phi^\dagger\phi)^2$$

$$\phi = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v+h \end{pmatrix} \qquad \phi^\dagger\phi = \frac{1}{2}(v+h)^2$$

mass term $\propto h^2$

$$\phi^\dagger\phi \to h^2$$

$$(\phi^\dagger\phi)^2 = \frac{1}{4}(v+h)^4 \to \frac{1}{4}\frac{4!}{2!2!}v^2h^2 = \frac{3}{2}v^2h^2$$

$$\text{mass term} = -\frac{\lambda v^2}{2} h^2 + \frac{3}{2}\lambda v^2 h^2 = \lambda v^2 h^2 = \frac{1}{2}m_h^2 h^2$$

$m_h^2 = 2v^2\lambda$ but λ is free parameter

Higher order EWK corrections plus Precision measurements gave good prediction of Higgs mass prior to discovery.

Fermion Masses terms $\bar{f}f$ violate $SU_L(2)$ symmetry. Fermion masses come from coupling to Higgs.

$$\mathcal{L}_e = -G_e\left[(\bar{\nu},\bar{e})_L\begin{pmatrix}\phi^+\\ \phi_0\end{pmatrix}e_R + \bar{e}_R(\phi^-,\phi_0)\begin{pmatrix}\nu\\ e\end{pmatrix}_L\right]$$

G_e ↑ free parameter

After spontaneous symmetry breaking (SSB) mass terms come from

$$\phi = \frac{1}{\sqrt{2}}\begin{pmatrix}0\\ v+h\end{pmatrix}$$

(other 3 ϕ can be removed by choice of gauge)

$$\mathcal{L}_e^{SSB} = -\frac{G_e v}{\sqrt{2}}\bar{e}e - \frac{G_e}{\sqrt{2}}\bar{e}eh = -m_e\bar{e}e - \left(\frac{m_e}{v}\right)\bar{e}eh$$

Higgs coupling and mass. Higgs coupling $\propto m_e$

neutrino has no ν_R and is massless.

Quark masses need charge conjugate field

$$\phi_c = -i\tau_2\phi^* = \begin{pmatrix}-\bar{\phi}^0\\ \phi^-\end{pmatrix} \to \frac{1}{\sqrt{2}}\begin{pmatrix}v+h\\ 0\end{pmatrix} \quad Y=-1$$

$$\Delta\mathcal{L}_q = -G_d^{ij}(\bar{u}_i,\bar{d}_i)_L\begin{pmatrix}\phi^+\\ \phi_0\end{pmatrix}d_{jR} - G_u^{ij}(\bar{u}_i,\bar{d}_i)_L\begin{pmatrix}-\bar{\phi}^0\\ \phi^-\end{pmatrix}u_{jR}$$

+ hermitian conjugate

i,j = family indicies

matrices $G^{ij}_{u,d}$ allow for CKM matrix.

remaining free parameters are quark masses

SM Parameters:

$$\left.\begin{array}{l} g \\ \theta_W \\ v \end{array}\right\} \quad \alpha,\ G_F,\ M_Z$$

λ $\quad m_h$

12 fermion masses

(3 neutrino masses) not in SM

3 CKM angles

1 CKM phase

1 θ_{QCD} CP QCD vacuum parameter (neutron EDM)

21 free parameters (+ neutrino masses)

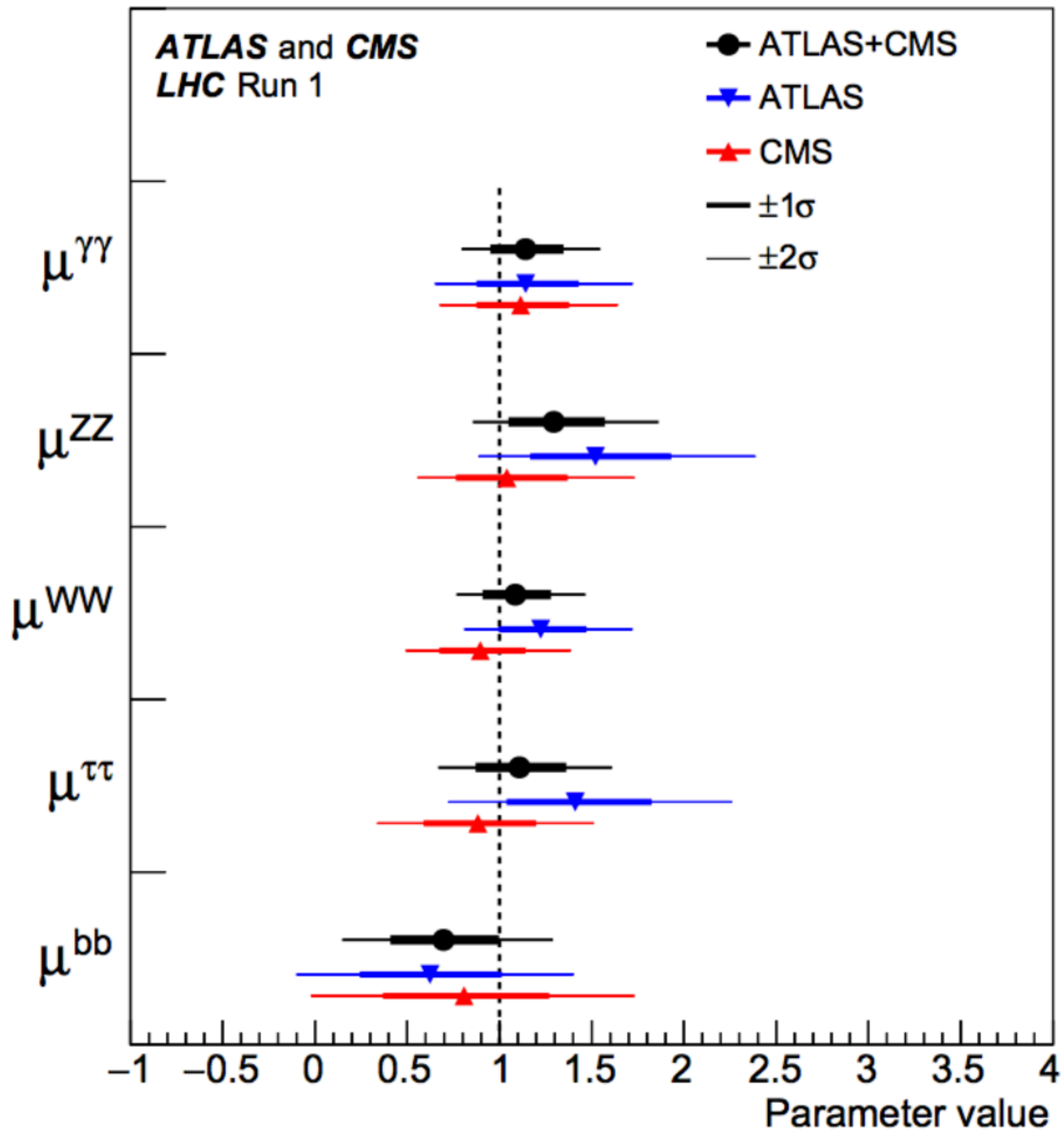


Figure 38: Summary of Higgs μ measurements, from [56].

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Lectures on the Theory of the Weak Interaction Michael E. Peskin

Higgs vacuum expectation value (v) and
Vacuum energy

$$V(\phi) = \mu^2(\phi^+\phi) + \lambda(\phi^+\phi)^2$$

$$\langle\phi\rangle_{vac} = \sqrt{\frac{-\mu^2}{2\lambda}} \equiv \frac{v}{\sqrt{2}}$$

$$V_{min} = \mu^2\left(\frac{v^2}{2}\right) + \lambda\frac{v^4}{4} = -\lambda v^2\left[\frac{v^2}{2}\right] + \lambda\frac{v^4}{4}$$

$$= -\frac{\lambda v^4}{4}$$

$M_H^2 = 2v^2\lambda$ giving

$$V_{min} = -\frac{1}{8}M_H^2 v^2 \qquad v = 246\text{ GeV};\ M_H = 125\text{ GeV}$$

$$= -10^8\text{ GeV}^4 = -\frac{(10^4\text{ GeV})^4}{(\hbar c)^3}$$

Compared to measured vacuum energy density

$$\rho_{vac}c^2 = +\frac{(2\times10^{-3}\text{ eV})^4}{(\hbar c)^3}$$