Phys 450

Spring 2021

Lec #4 Symmetries II

The strong interaction (quantum chromodynamics) is flavor independent. The strong nuclear force (pion exchange) is isospin invariant, the interaction between nucleons (p, n) are the same.

$|N\rangle = |\tfrac{1}{2}, m\rangle \qquad m = +\tfrac{1}{2}$ proton or $\begin{pmatrix}1\\0\end{pmatrix}$

$\qquad\qquad\qquad\qquad m = -\tfrac{1}{2}$ neutron or $\begin{pmatrix}0\\1\end{pmatrix}$

Form an isospin doublet.

$|\pi\rangle = |1, m\rangle \qquad m = +1, \pi^+$

$\qquad\qquad\qquad m = 0, \pi^0$

$\qquad\qquad\qquad m = -1, \pi^-$

We can use the symmetry to find relations between cross sections.

For example (1)

$$\frac{\sigma(pp \to \pi^+ d)}{\sigma(np \to \pi^0 d)}$$

the deuteron d has isospin = 0

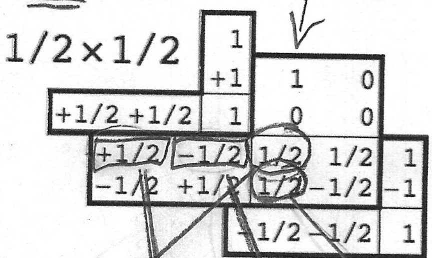
Example 1

$$\underset{\sim}{\tfrac{1}{2}} \otimes \underset{\sim}{\tfrac{1}{2}} = \underset{\sim}{1} + \underset{\sim}{0}$$

I boxed spinor states ⇒

I circled - C.G. coefficients →

Note: A square-root sign is to be understood



$Y_1^0 =$

Y_1^1

symmetric

$$|1,0\rangle = \frac{1}{\sqrt{2}}\left|\tfrac{1}{2},+\tfrac{1}{2}\right\rangle_1\left|\tfrac{1}{2},-\tfrac{1}{2}\right\rangle_2 + \frac{1}{\sqrt{2}}\left|\tfrac{1}{2},-\tfrac{1}{2}\right\rangle_1\left|\tfrac{1}{2},+\tfrac{1}{2}\right\rangle_2$$

$$-\tfrac{1}{2}+\tfrac{1}{2}=0$$

$$|0,0\rangle = \frac{1}{\sqrt{2}}\left|\tfrac{1}{2}\right\rangle\left|-\tfrac{1}{2}\right\rangle - \frac{1}{\sqrt{2}}\left|-\tfrac{1}{2}\right\rangle\left|\tfrac{1}{2}\right\rangle$$

antisymmetric

$$\langle 1,0|0,0\rangle = 0$$

C.G. Tables can be read the other way

Note: A square-root sign is to be understood

$1/2 \times 1/2$		1	1	0	1
		+1	0	0	−1
+1/2	+1/2	1			
+1/2	−1/2		1/2	1/2	
−1/2	+1/2		1/2	−1/2	
−1/2	−1/2				1

$Y_1^0 =$

Y_1^1

$$\rightarrow \left|\tfrac{1}{2},\tfrac{1}{2}\right\rangle\left|\tfrac{1}{2},-\tfrac{1}{2}\right\rangle = \frac{1}{\sqrt{2}}\left|1,0\right\rangle + \frac{1}{\sqrt{2}}\left|0,0\right\rangle$$

$$\Rightarrow \left|\tfrac{1}{2},-\tfrac{1}{2}\right\rangle\left|\tfrac{1}{2},+\tfrac{1}{2}\right\rangle = \frac{1}{\sqrt{2}}\left|1,0\right\rangle - \frac{1}{\sqrt{2}}\left|0,0\right\rangle$$

$pp \sim |\tfrac{1}{2},\tfrac{1}{2}\rangle|\tfrac{1}{2},\tfrac{1}{2}\rangle = |1,1\rangle \sim \pi^+$

$np \sim |\tfrac{1}{2},-\tfrac{1}{2}\rangle|\tfrac{1}{2}+\tfrac{1}{2}\rangle = \frac{1}{\sqrt{2}}|1,0\rangle^{\sim \pi^0} + \frac{1}{\sqrt{2}}|0,0\rangle^{\sim \eta}$

reading C.G. table horizontal to vertical

$\langle pp|\hat{m}|\pi^+ d\rangle = M_1$

a Isospin I=1 amplitude can only depend on I.

$\langle np|\hat{m}|\pi^0 d\rangle = \frac{1}{\sqrt{2}} M_1$

Ratio of σ is $\dfrac{|M_1|^2}{|\frac{1}{\sqrt{2}} M_1|^2} = 2$

<u>example (2)</u>

$\pi + N \to \pi' + N'$ pion nucleon scattering

find relations from C.G. coefficients

$\pi : |1, m\rangle \quad I = 1$

$N : |\tfrac{1}{2}, m\rangle \quad I = \tfrac{1}{2}$

Reading C.G. Table (the other way)

(c) $\pi^+ p \to \pi^- p$

(f) $\pi^- p \to \pi^0 n$

$$|\pi^- p\rangle = |1,-1\rangle |\tfrac{1}{2}, \tfrac{1}{2}\rangle = \sqrt{\tfrac{1}{3}}\, |\tfrac{3}{2}, -\tfrac{1}{2}\rangle - \sqrt{\tfrac{2}{3}}\, |\tfrac{1}{2}, -\tfrac{1}{2}\rangle$$

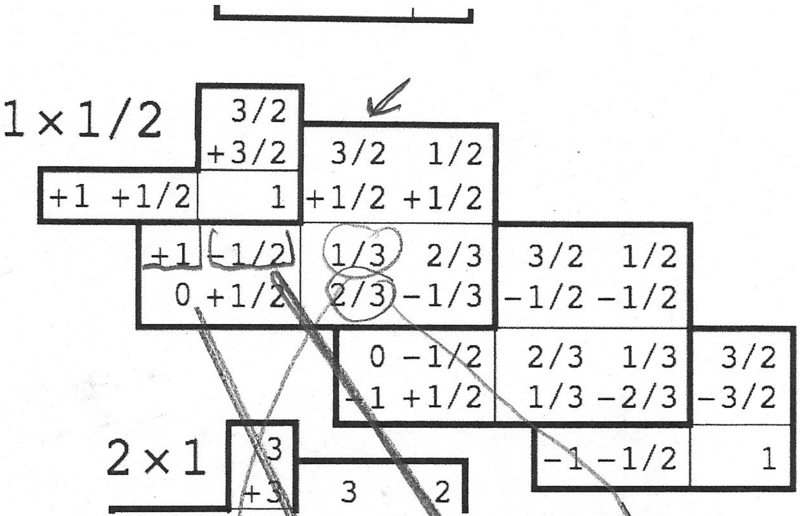
$$|\pi^0 n\rangle = |1,0\rangle |\tfrac{1}{2}, -\tfrac{1}{2}\rangle = \sqrt{\tfrac{2}{3}}\, |\tfrac{3}{2}, -\tfrac{1}{2}\rangle + \tfrac{1}{\sqrt{3}}\, |\tfrac{1}{2}, -\tfrac{1}{2}\rangle$$

$$\langle \pi^- p | \hat{M} | \pi^- p\rangle = \frac{1}{\sqrt{3}} \underbrace{M_{3/2}}_{I = 3/2} + \sqrt{\tfrac{2}{3}} \underbrace{M_{1/2}}_{I = 1/2}$$

$$\langle \pi^- p | \hat{M} | \pi^0 n\rangle = \frac{\sqrt{2}}{3} M_{3/2} - \frac{\sqrt{2}}{3} M_{1/2}$$

example 2

$$\underset{\sim}{1} \otimes \underset{\sim}{\tfrac{1}{2}} = \underset{\sim}{\tfrac{3}{2}} \oplus \underset{\sim}{\tfrac{1}{2}}$$



$$\left|\tfrac{3}{2}, \tfrac{1}{2}\right\rangle = \frac{1}{\sqrt{3}} |1,1\rangle \underbrace{\left|\tfrac{1}{2}, -\tfrac{1}{2}\right\rangle}_{1-\frac{1}{2}=\frac{1}{2}} + \sqrt{\tfrac{2}{3}}\, |1,0\rangle \underbrace{\left|\tfrac{1}{2}, \tfrac{1}{2}\right\rangle}_{0+\frac{1}{2}=+\frac{1}{2}}$$

$$\left|\tfrac{1}{2}, \tfrac{1}{2}\right\rangle = \sqrt{\tfrac{2}{3}}\, |1,1\rangle \underbrace{\left|\tfrac{1}{2}, -\tfrac{1}{2}\right\rangle}_{1-\frac{1}{2}=+\frac{1}{2}} - \sqrt{\tfrac{1}{3}}\, |1,0\rangle \underbrace{\left|\tfrac{1}{2}, \tfrac{1}{2}\right\rangle}_{0+\frac{1}{2}=\frac{1}{2}}$$

$$\left\langle \tfrac{3}{2}, \tfrac{1}{2} \middle| \tfrac{1}{2}, \tfrac{1}{2} \right\rangle = 0$$

Parity P $\vec{r} \to -\vec{r}$ spatial inversion

$t \to t$ $P^2 = 1$

Weak interaction violate parity conservation

Angular momentum: $Y_\ell^m(\theta,\phi) \to Y_\ell^m(\pi-\theta, \pi+\phi) = (-1)^\ell Y_\ell^m(\theta,\phi)$

Parity is a multiplicative quantum number

the π has spin = 0 and odd intrinsic parity

Slow absorption of π^- by deuterium
($\ell = 0$ initial state) Perkins

$\pi^- + d \to n + n$

J initial = 1 + 0 = 1

two identical neutrons in final state must be antisymmetric.

S = 1, triplet spin (aligned) state is symmetric $(-1)^{S+1} = +1$

S = 0, singlet spin state is antisymmetric $(-1)^{S+1} = -1$

overall symmetry of 2 neutron state is

$$-1 = (-1)^\ell (-1)^{S+1}$$

$$(-1)^{\ell+S} = +1 \quad \ell + S \text{ even}$$

to get $\vec{j} = 1$ in final state,

$$|l - s| \le j \le l + s$$

$s = 0 \Rightarrow l = 1$

$s = 1 \Rightarrow l = 2, 1, 0$

$\Big\}$ only $l = 1, s = 1$ gives $l + s$ even

$n + n$ final state is $l = 1,\ S = 1$

<u>intrinsic</u> parity of n, p are $+1$

final state <u>parity</u> is $(-1)^l = -1$ odd

initial state parity is $(+1$ for deuteron$) \times ($parity of pion$)$

$\therefore$ pion has <u>odd</u> intrinsic parity.

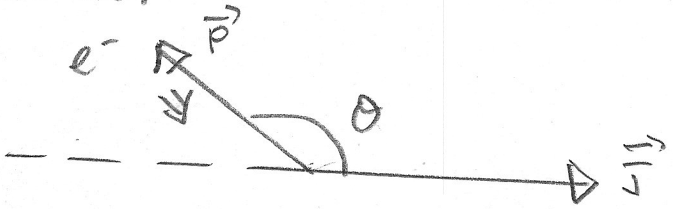
π: $J^P = 0^-$ <u>pseudoscalar</u> mesons

$\not{P}$ in weak interaction   Wu et al. 1957

^{60}Co @ $T = 0.01K$ inside solenoid, field $\vec{B}$

$^{60}Co\,(J=5) \rightarrow {}^{60}Ni^{*}\,(J=4) + e^- + \bar{\nu}_e$
$\hookrightarrow \gamma$

angular distribution of γ measures Co polarization.

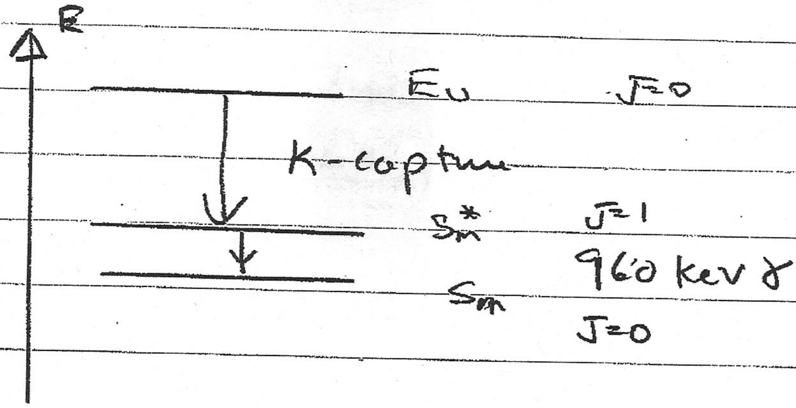


In 100% polarization limit

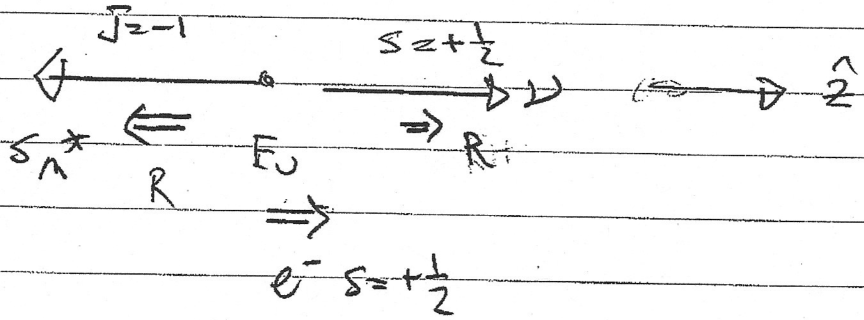
$$\frac{dn}{d\Omega}(\theta) \propto 1 - \frac{|\vec{p}|}{E}\cos\theta \qquad \cos\theta = \frac{\vec{p}\cdot\vec{J}}{|\vec{p}||\vec{J}|}$$

4 <u>neutrino helicity</u> Goldhaber, Grodzins, Sunyar (1958)

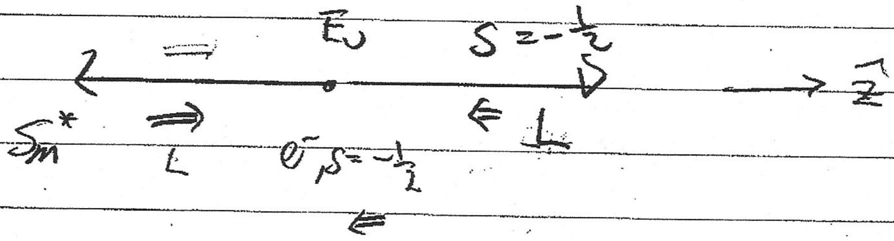
(i) ^{152}Eu undergoes K-capture to $^{152}Sm^*$ $(J=1)$



$$Eu + e^- \rightarrow Sm^* + \nu_e$$

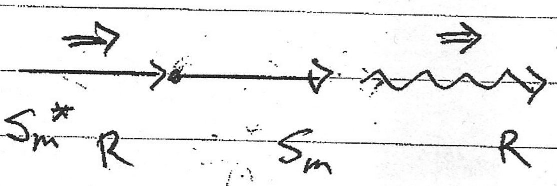


or



Sm^* has same helicity as neutrino

(ii) photon emitted in Sm direction has same polarization as Sm^*



(iii) resonance scattering $\gamma + {}^{152}Sm \to {}^{152}Sm^* \to \gamma + {}^{152}Sm$

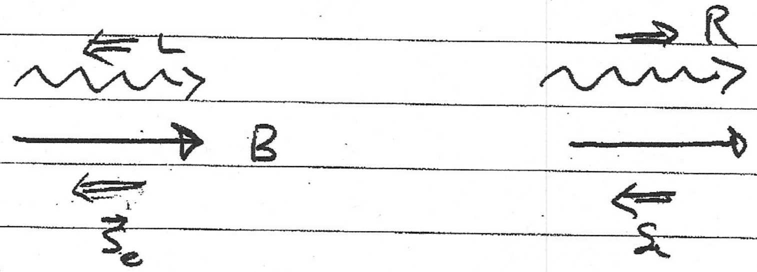
photon $E_\gamma > 960\,\text{keV}$ to allow for Sm^* recoil

Automatically selects forward photon

(iv) polarization of γ's

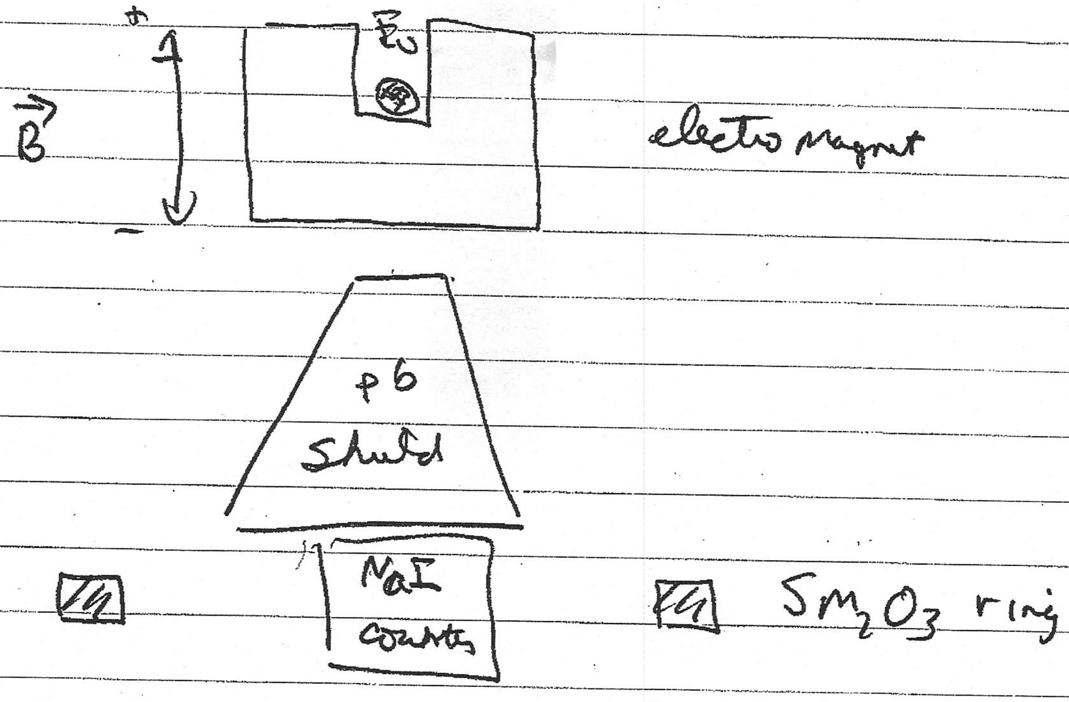
magnetized iron preceding Sm absorber

allows absorption of γ's due to spin flip.



transmission

absorption by spin flip.

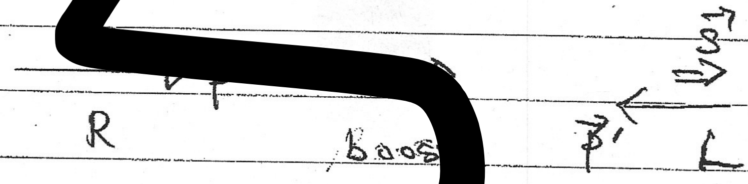


$$\delta = \frac{N_- - N_+}{\frac{1}{2}(N_- + N_+)} = +0.017 \pm 0.003$$

neutrino is left handed.

5 Chirality [redacted] factor

helicity is [redacted] invariant for massive particle.



Lorentz invariant [redacted]log is chirality.

$$|R\lambda\rangle = [redacted]\left[(1+v)|+H\rangle + (1-v)|-H\rangle\right]$$

v [redacted] P/E

π^- [redacted]ay $\Gamma(\pi^- \to \mu^- \bar{\nu}_\mu) \sim 100\%$

$\Gamma(\pi^- \to e$ [redacted] $\nu_\mu) \sim 10^{-4}$

$\Gamma \propto (1-v)$

muon momentum $p = \dfrac{m_\pi^2 - m_\mu^2}{2m_\pi}$

Helicity & Chirality

helicity eigenstates have spin aligned, antialigned with momentum

$$|{+}H\rangle: \quad \xrightarrow{\vec{p}} \;\; \Rightarrow \vec{s} \qquad \overset{\text{parity}}{\underset{\hat{P}}{\leadsto}} \qquad |{-}H\rangle: \quad \xleftarrow{\vec{p}} \;\; \Rightarrow \vec{s}$$

right handed   left handed

For massless particles, helicity is Lorentz invariant. Not so for particles with mass, as a boost can be found to reverse $\vec{p}$.

Chiral eigenstates are Lorentz invariant

They can be written as

right chiral $|R\rangle = A_R \left[(1+v)^{1/2} |{+}H\rangle + (1-v)^{1/2} |{-}H\rangle \right]$

left chiral $|L\rangle = A_L \left[(1-v)^{1/2} |{+}H\rangle - (1-v)^{1/2} |{-}H\rangle \right]$

evidently for massless particles, chirality = helicity

Parity violating weak interactions produce chiral eigenstates

ν is produced as $|L\rangle$ $\bar{\nu}$ as $|R\rangle$

e^- is produced as $|L\rangle$ e^+ as $|R\rangle$

phase space
$$\frac{dN}{dE} = \text{const}\; p^2 \frac{dp}{dE}$$

$\frac{d^3p}{E}$ is Lorentz invariant phase space

(will come into Fermi's golden rule)

$$E = p + \sqrt{p^2+m^2} = p + E_e$$

$$\frac{dE}{dp} = 1 + \frac{p}{\sqrt{p^2+m^2}} = \frac{E_e+p}{E_e}$$

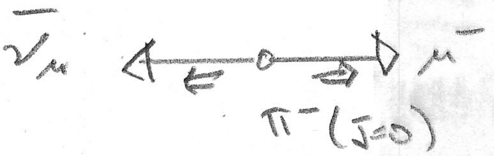
$$p^2 \frac{dp}{dE}\bigg|_{E=m_\pi} = \frac{p^2 E_e}{m_\pi} = \frac{1}{8 m_\pi^4}\,(m_\pi^2-m^2)^2\,(m_\pi^2+m^2)$$

$$\Gamma \propto \int p^2 \frac{dp}{dE}\,\delta(E-m_\pi)\,(1-v) = \frac{1}{4}\, m^2 \left(1-\left(\frac{m}{m_\pi}\right)^2\right)^2$$

$$\frac{\Gamma(\pi^- \to e^- \bar{\nu}_e)}{\Gamma(\pi^- \to \mu^- \bar{\nu}_\mu)} = \frac{m_e^2}{m_\mu^2}\,\frac{1}{\left(1-\left(\frac{m_\mu}{m_\pi}\right)^2\right)^2} \cong 1{,}3 \times 10^{-4}$$

example in helicity suppression of $\pi \to l\nu$ decay

lepton e or μ



angular momentum requires μ^- be in $|+H\rangle$ state

decay rate $\propto$ $|\langle L|+H\rangle|^2 = (1-v)$

$$\frac{\Gamma(\pi^- \to e^- \nu_e)}{\Gamma(\pi^- \to \mu^- \nu_\mu)} = \frac{(1-v_e)\times(\text{phase space})}{(1-v_\mu)\times(\text{phase space})}$$

kinematics:

$$(m_\pi, \vec{0}) = (p_\nu, -\vec{p}_\nu) + (E_e, \vec{p})$$

$\vec{p} = \vec{p}_\nu$ momentum conservation

$$m_\pi = E_e + p \qquad E_e = \sqrt{p^2+m^2}$$

given $\quad p = \frac{1}{2m_\pi}(m_\pi^2 - m^2)$

$$E_e = \frac{1}{2m_\pi}(m_\pi^2 + m^2)$$

$$v = \frac{p}{E_e} = \frac{m_\pi^2 - m^2}{m_\pi^2 + m^2}$$

$$1 - v = \frac{2m^2}{m_\pi^2 + m^2}$$

C : "charge parity" particle - antiparticle conjugation

strong, EM conserved weak $\not{C}$.

	p	$\bar{p}$
Q	e	$-e$
B	1	-1
μ	μ_p	$-\mu_p$
σ	$\frac{1}{2}\hbar$	$\frac{1}{2}\hbar$

	e^-	e^+
Q	$-e$	$+e$
L_e	$+1$	-1
μ	μ_e	$-\mu_e$
σ	$\frac{1}{2}\hbar$	$\frac{1}{2}\hbar$

$\mu_p = 2.79\, e\hbar/2m_pc$ $\qquad$ $\mu_e = e\hbar/2m_ec$

neutrino :

$\nu \xrightarrow[\Leftarrow]{} \vec{p} \quad \xrightarrow{CP} \quad \bar{\nu} \xrightarrow[\Rightarrow]{} \vec{p}$

left-handed ν $\qquad$ right handed $\bar{\nu}$

example of C conservation in EM interaction

EM coupling $\quad j_\mu A^\mu \xrightarrow[C]{} -j_\mu C^\dagger A^\mu C$

EM current changes sign

therefore $C|\gamma\rangle = -|\gamma\rangle$ odd

$\pi^0 \to \gamma\gamma$ EM decay implies $C|\pi^0\rangle = +|\pi^0\rangle$

$$\frac{Br(\pi^0 \to 3\gamma)}{Br(\pi^0 \to 2\gamma)} < 4 \times 10^{-7}$$

CPT conservation required by QFT

CPT theorem. neutron dipole moment ($\cancel{T}$)

$d_n = (0.0 \pm 1.1)\,10^{-26}\,e\cdot cm$

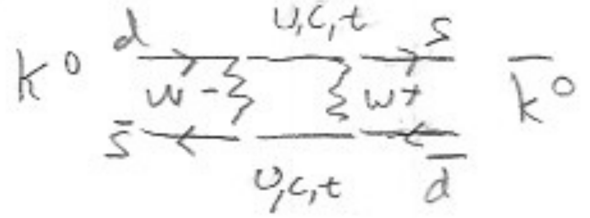
also spin-statistics theorem

<u>Weak interaction violate CP</u> ($\cancel{CP}$)

$$\pi^+ \to \mu^+ \nu_\mu \xrightarrow{CP} \pi^- \to \mu^- \bar{\nu}_\mu$$

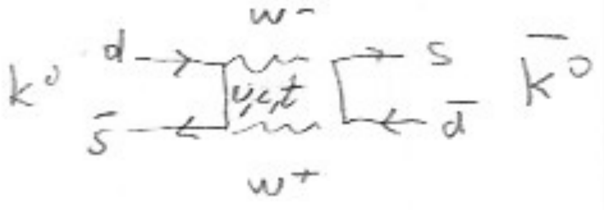
but $\cancel{CP}$ by $K_0 \cdot \bar{K}_0$ mixing

dominate by CKM matrix



charm quark

$$\Delta m = \frac{G^2}{4\pi} m_K f_K^2 m_c^2 \cos^2\theta_c \sin^2\theta_c$$



$f_K = 1.2\, m_\pi$

Kaon decay constant

<u>CP</u> mass eigenstate

$\Delta m = 3.49 \times 10^{-12}$ MeV

$$|K_S\rangle = \frac{1}{\sqrt{2}}\left(|K^0\rangle + |\bar{K}^0\rangle\right) \quad CP = +1$$

$$|K_L\rangle = \frac{1}{\sqrt{2}}\left(|K^0\rangle - |\bar{K}^0\rangle\right) \quad CP = -1$$

$\tau_S = 0.893 \times 10^{-10}$ s $K_S \to 2\pi$

$\tau_L = 0.517 \times 10^{-7}$ s $K_L \to 3\pi$

} difference due to Q decay (phase space)

CP̸ $K_L \to 2\pi$ observed

denote CP eigenstates as K_1 (CP+) K_2 (CP−)
physical states are

$$|K_L\rangle = \frac{1}{\sqrt{1+|\varepsilon|^2}}\left[|K_2\rangle + \varepsilon|K_1\rangle\right]$$

$$|K_S\rangle = \frac{1}{\sqrt{1+|\varepsilon|^2}}\left[|K_1\rangle - \varepsilon|K_2\rangle\right]$$

ε is small, CP̸ parameter $\varepsilon = 2.3\times10^{-3}$