phys 450

Spring 2021

Ch 5 - Bound states

Review of H-atom

Non-relativistic $V(r) = \frac{-e^2}{r} = -\frac{\alpha \hbar c}{r}$

quantum number $n, l, m_l\ (s=\frac{1}{2}), m_s$

degeneracy $\underset{\text{spin}}{\underset{\uparrow}{2}} n^2$ accidental degeneracy of $\frac{1}{r}$ potential

relativistic corrections (Dirac equation)

$$\hat{H} = mc^2 + \frac{\hat{p}^2}{2m} - \underbrace{\frac{\hat{p}^4}{8m^3c^2}}_{\hat{H}_K} + \underbrace{\frac{e}{mc}\vec{S}\cdot\vec{B}'}_{\hat{H}_{SO}} + \underbrace{\frac{\hbar^2}{8mc^2}\nabla^2 V}_{\hat{H}_D \text{ Darwin}^*}$$

e^- Zitterbewegung

$\vec{B}'$ is internal B due to proton current in e^- rest frame

in 1st order perturbation theory

only $l=0$ states

$$\hat{H}_D = \frac{\pi}{2}(mc^2)(Z\alpha)^4\left(\frac{a_0}{Z}\right)^3\delta^3(\vec{r})$$

$$E^{(1)}_{D\,n,0} = \langle \hat{H}^D\rangle_{n,0} = \frac{\pi}{2} mc^2 (Z\alpha)^4 \left(\frac{a_0}{Z}\right)^3 \underbrace{|\psi_{n,0}(0)|^2}_{\left(\frac{Z}{na_0}\right)^3\frac{1}{\pi}}$$

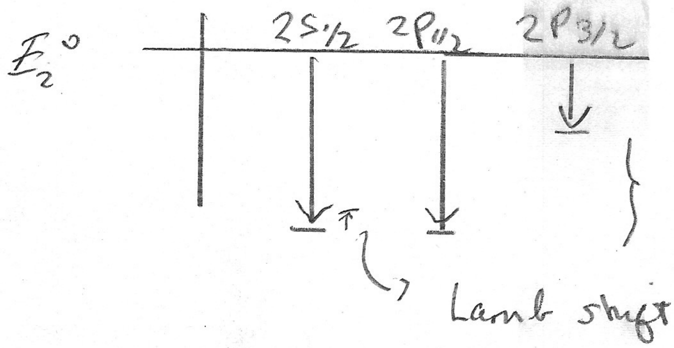
$$= \left(\frac{1}{2n^3}\right) mc^2 (Z\alpha)^4$$

* Charles Darwin grandson

lec #5-2

$$E_k^{(1)} + E_{so}^{(1)} = \frac{1}{8n^4}(Z\alpha)^4 mc^2 \left[3 - \frac{4n}{j+\frac{1}{2}}\right]$$

for $n=2$

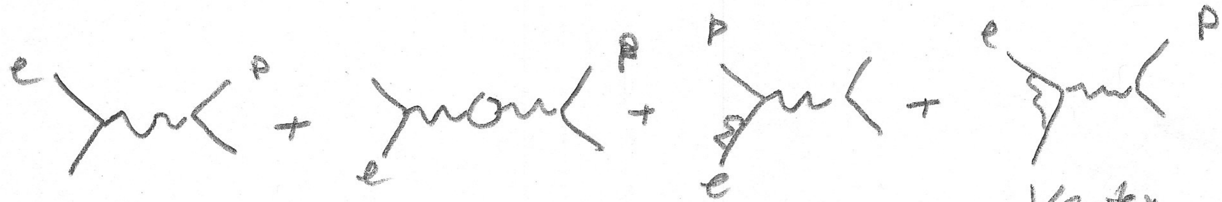


$$\Delta E_{fine} = \frac{4}{128}(mc^2)\alpha^4$$

$$\Delta\nu_f = \frac{\Delta E_{fine}}{h} = 10.9 GHz$$

$\Delta\nu_L = 1057 MHz$

QED corrections to H



Vacuum   e mass   Vertex anomalous magnetic moment

$-27 MHz$ $+1017 MHz$ $+68 MHz$

PRL 68 1120 (1992) 2S-2p splitting

$$\Delta E_L^{exp} = (1057.845 \pm 0.009) MHz$$

← 2p natural line width

$$\Delta E_L^{th} = (1057.874 \pm 0.018) MHz$$

↑

proton radius

Positronium (e⁺e⁻ bound state)

$|\psi\rangle = (\text{space})(\text{spin})(\text{charge})$ "C-parity"   EM decay conserves C

Observe   ($^{2s+1}L_J$ notation)

para $^1S_0 \to 2\gamma$ $J=0$ (2 photon Bose symmetry) $C=(-1)^2=+1$

ortho $^3S_1 \to 3\gamma$ $J=1$ $C=(-1)^3=-1$

For positronium state $C = P \times (\text{spin exchange})$

P for fermion-antifermion pair $= -1$
P for boson-antiboson pair $= +1$ } QFT

so $C|\psi\rangle = (-1)^{s+1}(-1) = (-1)^s$

giving $C|^1S_0\rangle = +1\,|^1S_0\rangle$

$C|^3S_1\rangle = -1\,|^3S_1\rangle$

$C(\text{fermion antifermion pair}) = (-1)^{\ell+s}$ in general

$\tau(2\gamma) = \left[\frac{1}{2} mc^2 \alpha^5\right]^{-1} \cong 1.2\times 10^{-10}\,s$

$\tau(3\gamma) = \left[\left(\frac{2}{9\pi}\right)(\pi^2-9)\alpha^6 mc^2\right]^{-1} \cong 1.4\times 10^{-7}\,s$

extra factor α

<u>Baryon wave functions</u> $\psi = \underbrace{(\text{Space})(\text{spin})(\text{flavor})}_{\text{symmetric}}(\text{Color})_A$

lowest mass $\underset{\sim}{10}$ $(\ell = 0)$

$$\psi_{\underset{\sim}{10}} = \psi_S(\text{spin})\,\psi_S(\text{flavor})$$

$$|\Delta^+ : 3/2\ -\tfrac{1}{2}\rangle = \left(\frac{uud + udu + duu}{\sqrt{3}}\right)\left(\frac{\downarrow\downarrow\uparrow + \downarrow\uparrow\downarrow + \uparrow\downarrow\downarrow}{\sqrt{3}}\right)$$

lowest mass $\underset{\sim}{8}$

spin flavor

$$\psi_{\underset{\sim}{8}} = \frac{\sqrt{2}}{3}\left[\psi_{12}\phi_{12} + \psi_{23}\phi_{23} + \psi_{13}\phi_{13}\right]$$

antisymmetrized labels

Spin $\frac{1}{2}$ ψ_{13} : $|\tfrac{1}{2},\tfrac{1}{2}\rangle_{13} = (\uparrow\uparrow\downarrow - \downarrow\uparrow\uparrow)\frac{1}{\sqrt{2}}$

$|\tfrac{1}{2},-\tfrac{1}{2}\rangle_{13} = (\uparrow\downarrow\downarrow - \downarrow\downarrow\uparrow)\frac{1}{\sqrt{2}}$

not linearly independent

$$|\tfrac{1}{2},\pm\tfrac{1}{2}\rangle_{13} = \left(|\tfrac{1}{2},\pm\tfrac{1}{2}\rangle_{12} + |\tfrac{1}{2},\pm\tfrac{1}{2}\rangle_{23}\right)\frac{1}{\sqrt{2}}$$

$$\psi_{\underset{\sim}{8}} = \frac{\sqrt{2}}{3}\left[\psi_{12}\phi_{12} + \psi_{23}\phi_{23} + \frac{(\psi_{12}+\psi_{23})}{\sqrt{2}}\frac{(\phi_{12}+\phi_{23})}{\sqrt{2}}\right]$$

$$|\psi_{\underset{\sim}{8}}|^2 = \frac{2}{9}\left|\left[\psi_{12}\phi_{12}(1+\tfrac{1}{2}) + \psi_{23}\phi_{23}(1+\tfrac{1}{2}) + 1/2\,\psi_{12}\phi_{23} + 1/2\,\psi_{23}\phi_{12}\right]^2\right|^2$$

$$= \frac{2}{9}\left[2\left(1+\tfrac{1}{2}\right)^2\right] = 1$$

proton wave function with $S_z = +\frac{1}{2}$

$$|p: \tfrac{1}{2}, +\tfrac{1}{2}\rangle = \frac{\sqrt{2}}{3}\Big[\tfrac{1}{2}(\uparrow\downarrow\uparrow - \downarrow\uparrow\uparrow)(udu - duu)$$

$$+ \tfrac{1}{2}(\uparrow\uparrow\downarrow - \uparrow\downarrow\uparrow)(\underline{uud} - duu)$$

$$+ \tfrac{1}{2}(\uparrow\uparrow\downarrow - \downarrow\uparrow\uparrow)(\underline{uud} - duu)\Big]$$

$$= \frac{\sqrt{2}}{3}\left(\frac{1}{2}\right)\Big[uu\underline{d}(2\uparrow\uparrow\underline{\downarrow} - \uparrow\downarrow\uparrow - \downarrow\uparrow\uparrow)$$

$$+ u\underline{d}u(2\uparrow\underline{\downarrow}\uparrow - \downarrow\uparrow\uparrow - \uparrow\uparrow\downarrow)$$

$$+ \underline{d}uu(2\underline{\downarrow}\uparrow\uparrow - \uparrow\downarrow\uparrow - \uparrow\uparrow\downarrow)\Big)$$

$$= \frac{1}{3\sqrt{2}}\Big[2\,u\!\uparrow u\!\uparrow d\!\downarrow - u\!\uparrow u\!\downarrow d\!\uparrow - u\!\downarrow u\!\uparrow d\!\uparrow\Big]$$

+ 2 permutations

<u>magnetic moment</u>:

$$\mu_p = 3^{\swarrow\text{permutation}}\left(\frac{2}{3\sqrt{2}}\right)^2(\mu_u + \mu_u - \mu_d) \quad \text{first term}$$

$$+ \underset{\substack{\uparrow \\ 2^{nd}\ 2\text{ terms}}}{2\cdot 3}\left(\frac{1}{3\sqrt{2}}\right)^2(\mu_d) = \frac{1}{3}(2)(2\mu_u - \mu_d) + \frac{1}{3}\mu_d$$

$$= \frac{1}{3}\left[4\mu_u - \mu_d\right] \quad \text{and} \quad \boxed{\frac{\mu_n}{\mu_p} = -\frac{2}{3}}$$

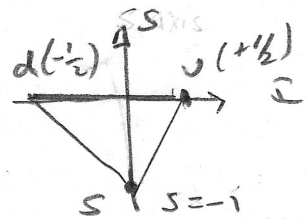
Quarkonium $(q\bar{q})$

$$V(r) = -\frac{4}{3}\frac{\alpha_s}{r} + F_0 r$$

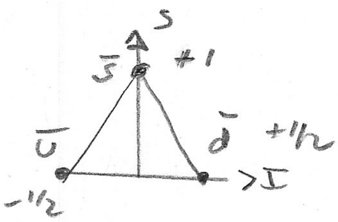
F_0 from $\Psi(c\bar{c})$ data $F_0 \cong 900$ MeV/fm

$t\bar{t}$ bound state doesn't exist because t decays too rapidly

$SU(3)_F$



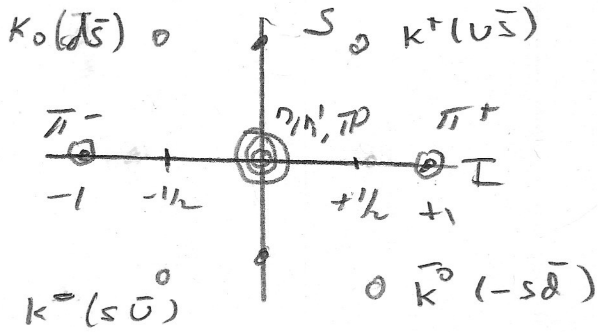
Quark



antiquark
inequivalent conjugate representation

SU(3) decomposition into irreps.

$$\underset{\sim}{3} \otimes \underset{\sim}{\bar{3}} = \underset{\sim}{8} \oplus \underset{\sim}{1} \qquad J=0 \text{ nonet}$$



scalar mesons ($J=0$)

$I=1$ $\quad \pi^+ = |1,1\rangle = -u\bar{d}$

$\pi^0 = |1,0\rangle = (u\bar{u} - d\bar{d})\frac{1}{\sqrt{2}}$

minus signs from q-bar in conjugate rep. of SU(2)

$\pi^- = |1,-1\rangle = d\bar{u}$

$I=0$ states mix to form physical

$\eta = (u\bar{u} + d\bar{d} - 2s\bar{s})\, 1/\sqrt{6}$

$\eta' = (u\bar{u} + d\bar{d} + s\bar{s})\, 1/\sqrt{2}$

Vector mesons:

$I=1$ $\quad \rho^+, \rho^0, \rho^-$ $\qquad I=\frac{1}{2}, S=1$ $\quad K^{*0}, K^{*+}$

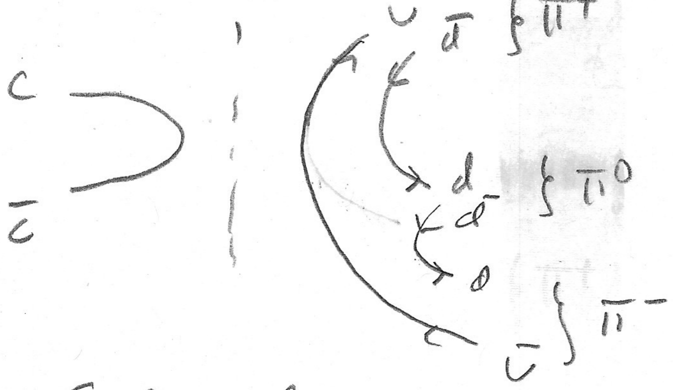
$I=\frac{1}{2}, S=-1$ $\quad K^{*-}, \bar{K}^{*}_{0}$

$I=0$ states

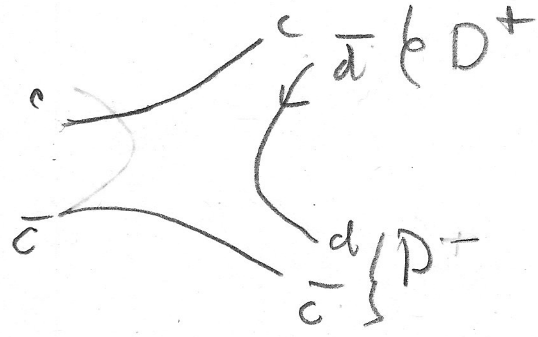
$\omega = (u\bar{u} + d\bar{d})\frac{1}{\sqrt{2}}$

$\phi = s\bar{s}$

OZI rule "3 gluon exchange"



Supressed
below open charm
threshold



open charm
decay

Lec #5–6

Discovery of charm

ψ is a vector meson with same spin/parity as photon

$$\Gamma(\psi) \cong 100\,\text{keV}$$
$$\Gamma(\psi') \cong 300\,\text{keV}$$
} extremely narrow resonance

$$e^+e^- \rightarrow \psi' \rightarrow \text{hadrons}$$

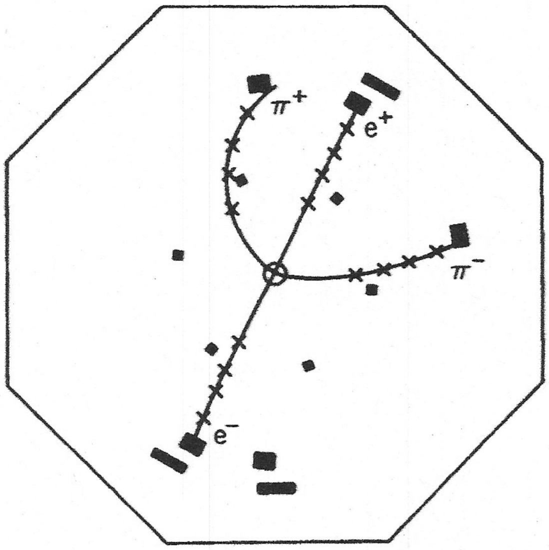


Figure 9.1. An example of the decay $\psi' \rightarrow \psi\pi^+\pi^-$ observed by the SLAC–LBL Mark I Collaboration. The crosses indicate spark chamber hits. The outer dark rectangles show hits in the time-of-flight counters. Ref. 9.5.

$$R = \frac{\sigma(e^+e^- \rightarrow \text{hadrons})}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)} \qquad \sigma_{had} = \sum_i \left| \quad e_i \quad \bar{q}_i \; q_i \right|^2$$

accessible quark flavors

CM energy

$$R(\sqrt{s} > 10\,\text{GeV}) = 3\sum e_i^2 = 3\left[\underset{u}{\left(\tfrac{2}{3}\right)^2} + \underset{d}{\left(\tfrac{1}{3}\right)^2} + \underset{s}{\left(\tfrac{1}{3}\right)^2} + \underset{c}{\left(\tfrac{2}{3}\right)^2} + \underset{b}{\left(\tfrac{1}{3}\right)^2}\right] = \frac{11}{3}$$

Color factor

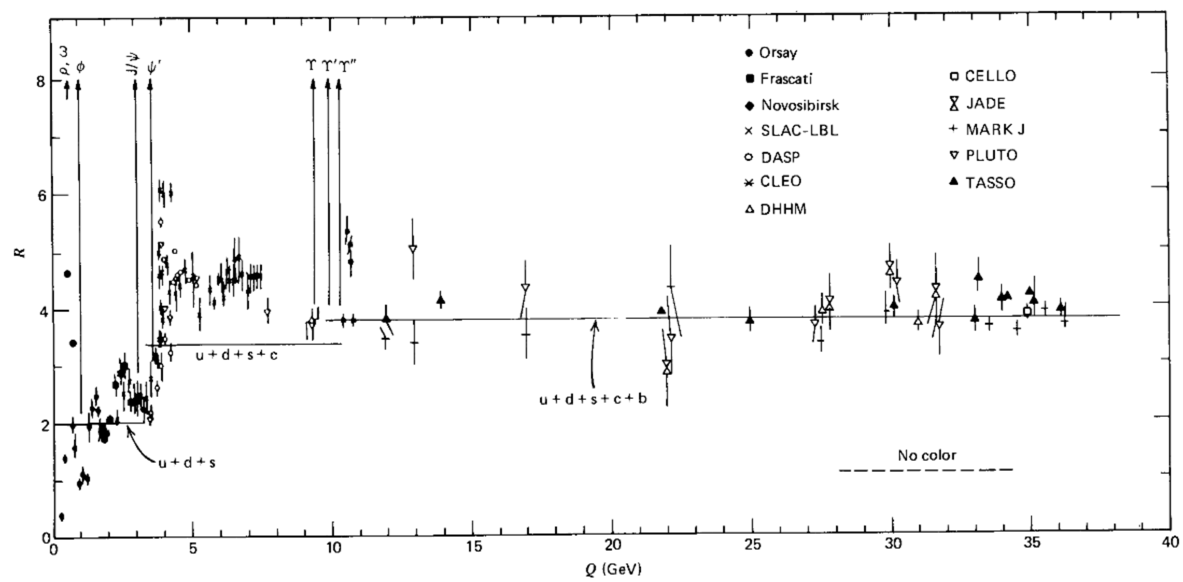


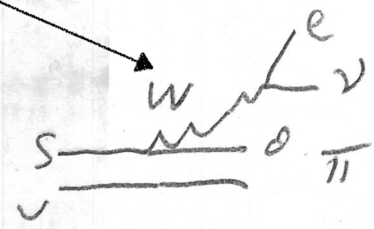
Fig. 11.3 Ratio R of (11.6) as a function of the total e^-e^+ center-of-mass energy. (The sharp peaks correspond to the production of narrow 1^- resonances just below or near the flavor thresholds.)

lec #5-7

charged current decay

τ(KL) = 5 x 10^(-8) s

$K_L^0 \to \pi^\pm e^\mp \nu_e$



neutral current decay $\boxed{K_L^0 \to \mu^+\mu^-}$

tiny due to GIM $B_r \cong 7 \times 10^{-9}$

neutral current

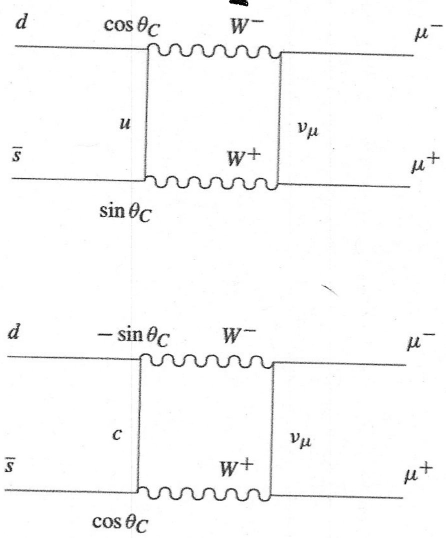


Figure 9.4. Two contributions to the decay $K_L^0 \to \mu^+\mu^-$ showing the factors present at the quark vertices. If only the upper contribution were present, the decay rate would be far in excess of the observed rate. The second contribution cancels most of the first. The cancellation would be exact if the c quark and u quark had the same mass. This cancellation is an example of the Glashow–Iliopoulos–Maiani mechanism.

GIM mechanism

Absence of flavor changing neutral currents.
predicted charm quark mass

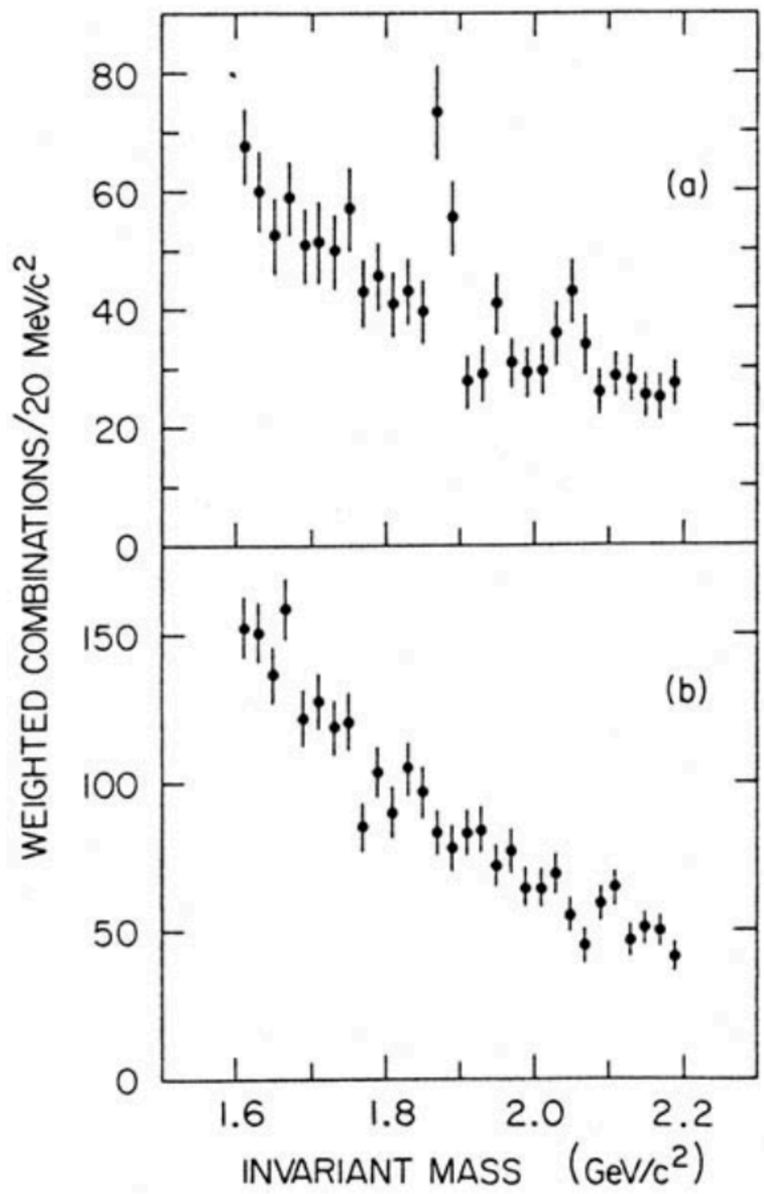


Figure 9.7. Invariant mass spectra for (a) $K^{\mp}\pi^{\pm}\pi^{\pm}$ and (b) $K^{\mp}\pi^{+}\pi^{-}$. Only the former figure shows a peak, in agreement with the prediction that D^{+} decays to $K^{-}\pi^{+}\pi^{+}$, but not $K^{+}\pi^{-}\pi^{+}$. (Ref. 9.12)

Lec #5-8

<u>Open charm</u> $e^+e^- \to \psi(3096) \to D, \bar{D}$

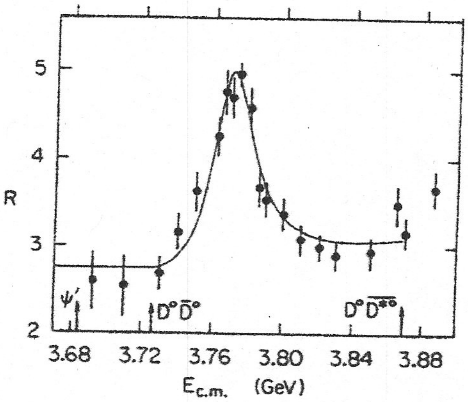


Figure 9.10. The $\psi(3772)$ resonance is broader than the $\psi(3096)$ and $\psi(3684)$ because it can decay into $D\overline{D}$. P. A. Rapidis *et al.*, (Ref. 9.14).

lec #5-9

$B^0 \bar{B}^0$ mixing

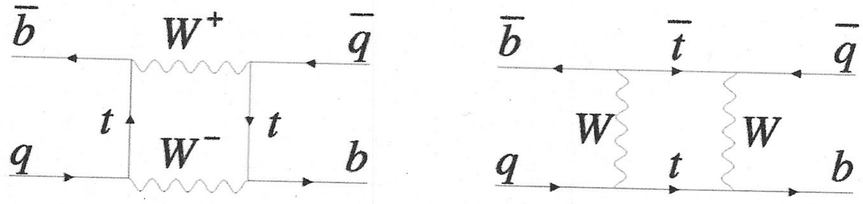


Figure 88.1: Dominant box diagrams for the $B_q^0 \to \overline{B}_q^0$ transitions ($q = d$ or s). Similar diagrams exist where one or both t quarks are replaced with c or u quarks.

$\Delta m \propto m_t^2$

$\Delta m_s \cong 18\ ps^{-1}$