Phys 450

Spring 2021

Lecture #6 : Feynman Rules

lifetime : $\frac{dN}{dt} = -\Gamma_{TOT} N$ constant decay probability per unit time

mean (proper) lifetime $\tau = \frac{1}{\Gamma_{TOT}}$

Partial widths : $B_r = \frac{\Gamma_i}{\Gamma_{TOT}}$ branching ratio

with $\Gamma_{TOT} = \sum_i \Gamma_i$

differential cross section :

$$\frac{d\sigma}{d\Omega} = \frac{1}{\mathcal{L}} \frac{dN}{d\Omega}$$

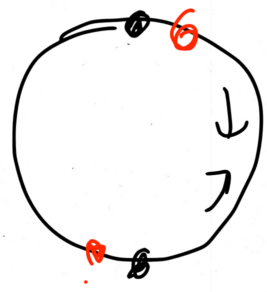
$\mathcal{L}$ = luminosity $\frac{\#\text{ particles}}{\text{area} \cdot \text{time}}$

for colliding beams

$$\mathcal{L} = f n \frac{N_1 N_2}{A}$$

A cross sectional area of beams

N_1, N_2 particles in bunches
n is number of bunches in ring
f is frequency of revolution around ring



"Golden" Rules

① decay of particle mass M in rest frame:

$$d\Gamma = S\frac{(2\pi)^4}{2M}|\mathcal{M}|^2\, d\Phi_n$$

S ≡ statistical factor for identical particles

$\mathcal{M}$: Lorentz invariant amplitude

$$d\Phi_n(\tilde{P};\tilde{p}_i) = (2\pi)^4\,\delta^4\left(\tilde{P}-\sum \tilde{p}_i\right)\prod_{i=1}^{n}\frac{d^3p_i}{(2\pi)^3 2E_i}$$

$\tilde{P} = (M, \vec{0})$

i ≡ final state label

Lorentz invariant phase space

② $$d\sigma = \frac{(2\pi)^4|\mathcal{M}|^2}{4\sqrt{(\tilde{p}_1\cdot\tilde{p}_2)^2 - m_1^2 m_2^2}}\, d\Phi_n(\tilde{p}_1+\tilde{p}_2\,;\,p_i)$$

final state

In the C.M. frame,

$$\sqrt{(\tilde{p}_1\cdot\tilde{p}_2)^2 - m_1^2 m_2^2} = p_1^{cm}\sqrt{s}$$

Mandelstam variable

$$\sqrt{s} = \left(E_1^{cm} + E_2^{cm}\right)$$

2 Body decay $M \to m_1 + m_2$

M rest frame

$$\vec{p} \longleftarrow \underset{M}{\circ} \longrightarrow -\vec{p}$$

$$d\Gamma = \frac{S}{2M} 2\pi^4 \delta^4(\tilde{P} - \tilde{P}_1 - \tilde{P}_2) |m|^2 \frac{d^3p_1}{(2\pi)^3 2E_1} \frac{d^3p_2}{(2\pi)^3 2E_2}$$

integrate over d^3p_2 using δ^3

$$\delta^{(4)}(\ ) = \delta^{(0)}(M - E_1 - E_2)\, \delta^{(3)}(-\vec{p}_1 - \vec{p}_2)$$

gives 3 momentum conservation $\vec{p}_2 = -\vec{p}_1 \equiv -\vec{p}$

$$\frac{d\Gamma}{d\Omega} = \frac{S}{2M} \frac{1}{(2\pi)^2} |m|^2 \int p^2 dp \frac{\delta^{(0)}(M - E_1 - E_2)}{4E_1E_2}$$

Total $E = E_1 + E_2 = \sqrt{m_1^2 + p^2} + \sqrt{m_2^2 + p^2}$

To use $\delta^{(0)}$ must change variables to E.

$$\frac{dE}{dp} = \frac{p}{E_1} + \frac{p}{E_2} = \frac{p(E_1+E_2)}{E_1E_2} = \frac{pE}{E_1E_2}$$

$$\frac{p^2dp}{4E_1E_2} = \frac{1}{4}\frac{p}{E} dE$$

integrate over $\delta^{(0)}(M - E)$ to get

$$\boxed{\frac{d\Gamma}{d\Omega} = \frac{S}{32\pi^2} \frac{p}{M^2} |m|^2}$$

P comes from Energy conservation:

$$m = \sqrt{m_1^2+p^2} + \sqrt{m_2^2+p^2}$$

$$\left(m - \sqrt{m_1^2+p^2}\right)^2 = m_2^2+p^2$$

$$m^2 - 2m\sqrt{m_1^2+p^2} + m_1^2 + \cancel{p^2} = m_2^2 + \cancel{p^2}$$

$$\left(m^2+m_1^2-m_2^2\right)^2 = \left(m_1^2+p^2\right)4m^2$$

$$\left[m^2+(m_1-m_2)(m_1+m_2)\right]^2 - 4m_1^2m^2 = p^2$$

cross term $2m^2(m_1^2-m_2^2) - 4m_1^2m^2$

$$= -2m^2(m_1^2+m_2^2)$$

$$= -m^2(m_1-m_2)^2 - m^2(m_1+m_2)^2$$

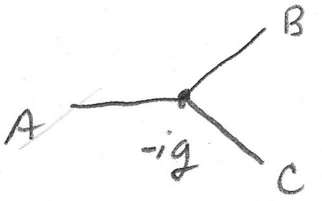
giving

$$p = \frac{1}{2m}\left[\left(m^2-(m_1+m_2)^2\right)\left(m^2-(m_1-m_2)^2\right)\right]^{1/2}$$

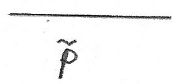
P.D.G. 48.17 or Griffiths 6.34

Feynman Rules for Toy A Theory

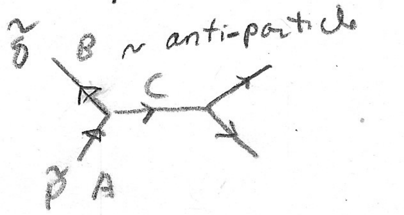
Vertex



$-ig$ factor



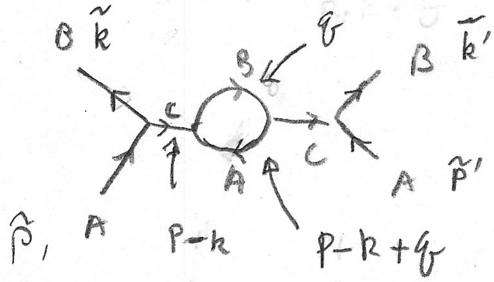
external line factor 1



internal line propagator factor

$$\frac{i}{(p-q)^2 - m_c^2}$$

4 momentum conserved at vertex



$$\int d^4q \left(\frac{i}{q^2 - m_B^2}\right)\left(\frac{i}{(p-k+q)^2 - m_c^2}\right)$$

integrate over internal 4-momentum

sign of $\tilde{q}$ is arbitrary

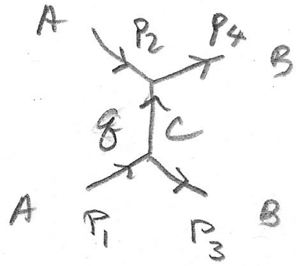
Loop diagram diverges as:

$$\int_{-\infty}^{\infty} d^4q \left(\frac{1}{q^4}\right) = \int d\Omega' \frac{q^3\,dq}{q^4} = \text{Const} \times \lim_{\Lambda \to \infty} \int_0^{\Lambda} \frac{dq}{q}$$

$$= \text{Const} \lim_{\Lambda \to \infty} \ln(\Lambda)$$

logrithmically divergent

Example Griffiths rules #6-5(a)



A + A → B + B

overall 4-momentum conservation

$p_1 + p_2 = p_3 + p_4$

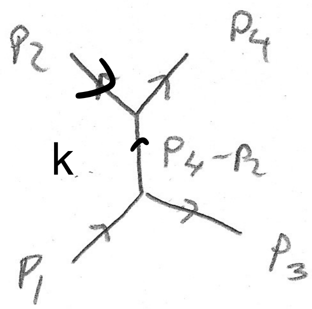
$$-i\mathcal{M} = (-ig)^2 \int d^4q \left(\frac{i}{q^2 - m_c^2}\right)(2\pi)^4 \delta^4 (p_1 - q - p_3)(2\pi)^4 (p_2 + q - p_4)$$

divide $(2\pi)^4\delta^4(\,)$ out to get

$q = p_4 - p_2$

$$-i\mathcal{M} = (-ig)^2 \frac{i}{(p_4 - p_2)^2 - m_c^2}$$

Or, impose 4-momentum at the start:

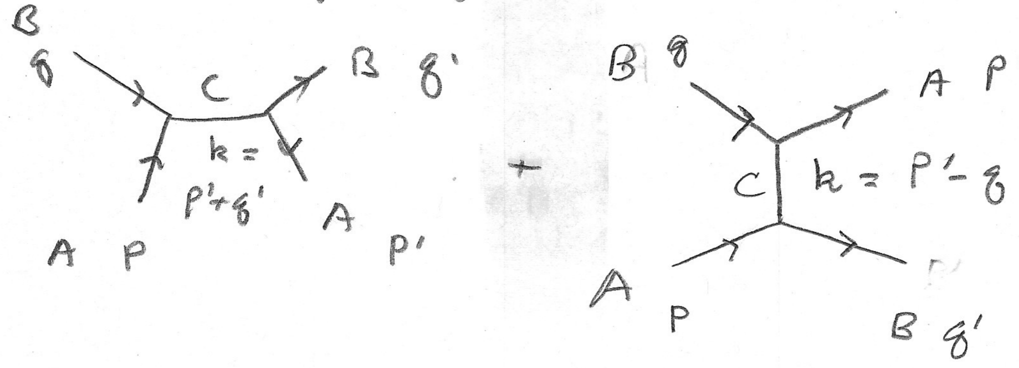


$$-i\mathcal{M} = (-ig)^2 \frac{i}{(p_4 - p_2)^2 - m_c^2}$$

Example

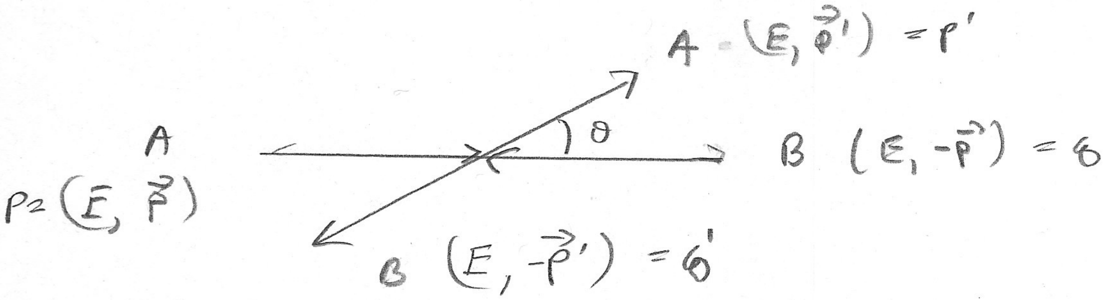
$A+B \to A+B$ in cm frame

$p+q = p'+q'$



take $m_A = m_B = m$, $m_C = 0$

$$-i\mathcal{M} = (-ig)^2\left[\frac{i}{(p'+q')^2} + \frac{i}{(p'-q)^2}\right]$$



$$\sqrt{(p\cdot q)^2 - m^4} = \left[(E^2+p^2)^2 - m^4\right]^{1/2} = 2Ep$$

$$d\sigma = |\mathcal{M}|^2 \frac{1}{4(2Ep)} (2\pi)^4 \delta^4(p+q-p'-q') \frac{d^3p'}{(2\pi)^3 2E'} \frac{d^3q'}{(2\pi)^3 2E'}$$

integrate over d^3q' using $\delta^{(3)}(\ )$, $\vec{q}' = -\vec{p}'$

$$d\sigma = |\mathcal{M}|^2 \frac{1}{4(2Ep)} \left(\frac{1}{2\pi}\right)^2 \left(\frac{1}{2E'}\right)^2 \delta^0(2E-2E')\, d\Omega\, p'^2 dp'$$

#6-7

Change variables $E'^2 = p'^2 + m^2 \Rightarrow E'dE' = p'dp'$

$$\int \delta^0(2E - 2E')\, p'^2 dp' = \frac{1}{2}\int \delta^0(E-E')\, Ep'\, dE' = \frac{1}{2}Ep$$

$$\frac{d\sigma}{d\Omega_{\vec{p}'}} = |m|^2 \frac{1}{4}\left(\frac{1}{2Ep}\right)\frac{1}{4\pi^2}\frac{1}{2}Ep$$

$$= |m|^2 \left(\frac{1}{2}\right)^8 \frac{1}{\pi^2}\frac{1}{E^4}$$

$$(p'+g')^2 = 4E^2 = 4(p^2+m^2)$$

$$(p'-g)^2 = \left|0, \vec{p}' + \vec{p}\right|^2 = -\left(2p^2 + 2p^2\cos\theta\right) = -2p^2(1+\cos\theta)$$

$$|m|^2 = g^4\left[\frac{1}{4E^2} - \frac{1}{2p^2(1+\cos\theta)}\right]^2$$

$$= \frac{g^4}{16}\left[\frac{1}{E^2} - \frac{1}{p^2\cos^2\theta/2}\right]^2$$