$$\equiv\alpha\int_0^\infty du\,u\,e^{-2u}, \tag{11}$$

the inequality being obvious, and the variable change in the last step is $u=t^\alpha$. The final integral is elementary and equals $\frac{1}{4}$, so that the first integral on the rhs of (9) is bounded from above by $\alpha/4$. Thence

$$\mathscr{E}(\alpha)<C(\alpha)^{-1}\times\left(\frac{\pi\alpha}{4}-\int_0^\infty dr\,r\int_0^{2\pi}d\phi\,e^{-2(r+r_0)^\alpha}|V(r,\phi)|\right). \tag{12}$$

Consequently, since $C(\alpha)$ is positive, one can always pick an α sufficiently small so as to allow the integral in (12) to dominate the $\pi\alpha/4$ term. This is because the value of this integral is essentially *independent* of α in this limit, the exponential approaching unit, making the rhs *negative*, or

$$E_0\leqslant\mathscr{E}(\alpha)<0.$$

Here, the first inequality follows from the Rayleigh–Ritz variational theorem. This, coupled with the condition (2), guarantees that the ground state E_0 is a bound level, Q.E.D.

We remark that, as stated initially, the variational proof would fail for any trial function of the form $\Phi(f(\sigma)r)$, including the Gaussian $e^{-\alpha r^2}$. In that case, the first integral on the rhs of (7) [introducing the new variable $\rho\equiv f(\alpha)r$] would become

$$\int_0^\infty dr\left(\frac{d\Phi(f(\alpha)r)}{dr}\right)^2 r=\int_0^\infty d\rho\,\rho\left(\frac{d\Phi(\rho)}{d\rho}\right)^2, \tag{13}$$

and thus really be independent of the variational parameter α, so that for a sufficiently shallow well $\mathscr{E}(\alpha)>0$ would result.

Finally, we mention that one can easily see that, for a potential that vanishes beyond a certain finite radius, the minimum value of $\mathscr{E}(\alpha)$ [obtained from $d\mathscr{E}(\alpha)/d\alpha=0$] approaches

$$c_1\frac{v^{7/2}}{\ln v}v^{c_2/v}e^{-c_3/v},\quad\text{as}\quad v\equiv\int_0^{2\pi}d\phi\int_0^\infty dr\,r|V(r,\phi)|\to 0,$$

where the c_is are positive constants. In other words, as expected from the upper-bound property of the variational energy $\mathscr{E}(\alpha)$, as the potential becomes arbitrarily weak 0^- is approached even *faster* than with the characteristic $-c_4e^{-c_5/v}$ behavior of the exact energy E_0, which can be proved[2] for the two-dimensional well. This latter exponential behavior is typical, for example, of the Bardeen–Cooper–Schriefer (BCS) theory of superconductivity. It is in marked contrast with the one-dimensional behavior[6] $E_0\to c_6v^2$. The readily verified fact that $e^{-c_5/v}$ has no Maclaurin (power) series expansion about $v=0$ means, in the BCS problem, that the "superconductive" phase is *not* obtainable from a perturbative series expansion about the "normal" phase as a reference or unperturbed state.

[1]D. ter Haar, *Selected Problems in Quantum Mechanics* (Academic, New York, 1964), p. 7.

[2]L. D. Landau and E. M. Lifshitz, *Quantum Mechanics* (Pergamon, New York, 1977), p. 163.

[3]E. N. Economou, *Green's Functions in Quantum Physics* (Springer-Verlag, New York, 1979), p. 68.

[4]D. L. Goodstein, *The State of Matter* (Prentice-Hall, Englewood Cliffs, NJ, 1975), p. 391.

[5]E. N. Economou and C. M. Soukoulis, Phys. Rev. B **28**, 1093 (1983); E. N. Economou, C. M. Soukoulis, and A. D. Zdetsis, Phys. Rev. B **30**, 1686 (1984).

[6]C. Cohen-Tannoudji, B. Diu, and F. Laloe, *Quantum Mechanics* (Wiley, New York, 1977), Vol. I, p. 89.

The EPR experiment, special relativity, and the distinction between effects and signals

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For over 50 years the Einstein–Podolsky–Rosen experiment[1] has generated considerable debate among physicists. Much of that discussion has centered on reconciling the instantaneous correlation of measurements over spacelike intervals with the principles of special relativity. Toward this end, some have felt the need to give up the concept of locality, implying that the correlation of measurements does not result from one measurement influencing another. In this note, I shall present an alternate interpretation in which the two measurements of the EPR experiment are considered to affect one another. I hope to show that such an interpretation is conceptually satisfying if viewed in the context of the distinction between "encoded" signals and other physical effects in special relativity.

Consider Bohm's formulation[2] of the EPR experiment. A source of electrons is placed between two Stern–Gerlach detectors whose measurements are made along the same transverse axis. The source emits pairs of electrons in the singlet state,

$$|\psi\rangle=(1/\sqrt{2})[(|+-\rangle-|-+\rangle)], \tag{1}$$

with the electrons moving in opposite directions, one toward each detector. After trips of arbitrary length (through vacuum), the electrons enter the detectors. Quantum theory predicts that whenever one detector measures the spin of one of the two electrons in the singlet pair as pointing up, the other will measure the spin of its electron as pointing down.

The observed correlation[3] of electron spins in EPR experiments with particular orientations of the detectors cannot be explained by "hidden variables" carried by the electrons prior to measurements; the existence of such quantities is proscribed by Bell's theorem.[4] (The proof of this is well known and need not be repeated here—Mermin[5] gives a particularly lucid discussion of this.)

If hidden variables are not allowed, can we accept a picture of the EPR wavefunction collapse in which the measurement of the spin of one of the particles uniquely determines the spin of the other without violating the principles of relativity? I believe we can. It is true that if one observer measures the spin of his electron as down the other will observe the spin of her electron as up. However, neither observer can control whether the spin of the electron is measured as up or down and thus neither observer can control the outcome of the measurement of the other observer. Each observer has a 50% chance of measuring the spin of the electron as up or down, period. As a result, it is quite impossible for the "EPR phenomenon" to be used to encode a message or send a signal at any velocity, superluminal or otherwise.

Now special relativity is full of "effects" with palpable physical consequences that may proceed through space at arbitrarily high velocity. The moving vertex of a closing pair of scissors or the traveling intersection point of two flashlight beams are two examples. Such effects cannot transfer information encoded by observers located along the points in space-time through which they travel. It seems reasonable that one should regard the EPR experiment or, for that matter, the collapse of *any* wavefunction, as an analogous phenomenon. The classical relativistic effects cited above cannot carry information due to the necessary positions of force-exerting observers or limits on rigidity—these effects are not "encodable." (Examples of noninformation-bearing effects traveling faster than c may be found even in classical electromagnetism—the phase velocity of light in certain media is greater than c but, as it is the group velocity with which the transfer of information is associated, this is not paradoxical.) In contrast, in the EPR experiment, the impossibility of signals arises from the statistical nature of the collapse of the wavefunction, but this distinction need not be a troubling one. So long as it is kept in mind that relativity does not forbid nonsignal-carrying effects from traveling at arbitrarily high velocity, the concept of the observations of one of the EPR detectors affecting the measurement of the other is a philosophically tractable one.

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[1] A. Einstein, B. Podolsky, and N. Rosen, Phys. Rev. **47**, 777 (1935).
[2] D. Bohm, *Quantum Theory* (Prentice-Hall, New York, 1951), Chap. 22.
[3] A. Aspect, P. Grangier, and G. Roger, Phys. Rev. Lett. **47**, 460 (1981); A. Aspect, P. Grangier, and G. Roger, Phys. Rev. Lett. **49**, 91 (1982); A. Aspect, J. Dalibard, and G. Roger, Phys. Rev. Lett. **49**, 1804 (1982).
[4] J. S. Bell, Physics **1**, 195 (1964).
[5] N. D. Mermin, Phys. Today **4**, 38 (1985).

Generalized coordinate representation in quantum mechanics

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In the original work of Podolsky,[1] the generalized coordinate representation in quantum mechanics results from a transformation of the coordinate system, i.e., a transformation from Cartesian coordinates to generalized coordinates. This is equivalent to a transformation of representation in quantum mechanics. Therefore, the generalized coordinate representation in quantum mechanics could be obtained from the theory of representations directly. The purpose of this note is to give this direct treatment on the generalized coordinate representation following Dirac's theory of representations.[2]

Suppose a set of generalized coordinates $x = (x^1, x^2, \ldots, x^n)$ forms a complete set of observables, then the complete set of the simultaneous eigenstates $\{|x\rangle\} = \{|x^1, x^2, \ldots, x^n\rangle\}$ can be used as the basis vectors for the representation. The eigenequations are

$$\hat{x}^j|x\rangle = x^j|x\rangle . \tag{1}$$

If the canonical momentum conjugate to the generalized coordinate x is denoted as $p = (p_1, p_2, \ldots, p_n)$, and the following Heisenberg commutation relations are assumed,

$$[\hat{x}^j, \hat{x}^k] = 0, \quad [\hat{p}_j, \hat{p}_k] = 0, \quad [\hat{x}^j, \hat{p}_k] = i\hbar\, \delta^j_k , \tag{2}$$

then an equation for the representative of the canonical momentum $\hat{p}_k$ can be obtained as

$$(x^j - x^{j\prime})\langle x|\hat{p}_k|x'\rangle = i\hbar\langle x|x'\rangle \delta^j_k . \tag{3}$$

This shows that $\langle x|\hat{p}_k|x'\rangle = 0$ if $j \neq k$ and $x^j \neq x^{j\prime}$, and

$$(x^k - x^{k\prime})\langle x|\hat{p}_k|x'\rangle = i\hbar\langle x|x'\rangle , \tag{4}$$

if $j = k$. Thus $\langle x|\hat{p}_k|x'\rangle = 0$ if $x^k \neq x^{k\prime}$, and $\langle x|\hat{p}_k|x'\rangle \to \infty$ if $x^k \to x^{k\prime}$. This means that the eigenstate $|x\rangle$ should be normalized to a δ function,

$$\langle x|x'\rangle = W^{-1}(x)\delta(x - x'), \tag{5}$$

where