Coherent State Worksheet
Intermediate Quantum 491

In class we showed that eigenstates of the harmonic oscillator creation operator (called coherent states)

$$\hat{a}|z\rangle = z|z\rangle$$
$$\langle z|\hat{a}^\dagger = z^*\langle z|$$

where z is a complex number $z = (y_0 + iq_0)/\sqrt{2}$.

We found that

$$|z\rangle = e^{-|z|^2/2}e^{z\hat{a}^\dagger}|0\rangle$$

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1. Find $\langle\hat{y}\rangle$, Δy and $\langle\hat{q}\rangle$, Δq for coherent states and show that coherent states have the minimum uncertainty product $\Delta x \Delta p$. Recall
$$\hat{y} = \hat{x}\sqrt{\frac{m\omega}{\hbar}}; \quad \hat{q} = \hat{p}\frac{1}{\sqrt{m\omega\hbar}}$$
2. Show that the probability distribution $|\langle n|z\rangle|^2$ is a Poisson with mean $|z|^2$.
3. Show that
$$|z\rangle = \exp\left(z\hat{a}^\dagger - z^*\hat{a}\right)|0\rangle$$
Hint: use the identity $e^{\hat{A}}e^{\hat{B}}e^{\frac{1}{2}[\hat{A},\hat{B}]} = e^{\hat{A}+\hat{B}}$ for $[\hat{A},\hat{B}]$ = a number (not an operator).
4. Show that $\exp\left(z\hat{a}^\dagger - z^*\hat{a}\right)$ is equal to a translation in momentum space times a translation in position space (up to an irrelevant phase factor).
5. show that
$$\langle E\rangle = \hbar\omega\left(|z|^2 + \frac{1}{2}\right)$$
and that
$$\Delta E = \hbar\omega|z|$$
and that therefore $\Delta E/\langle E\rangle \to 0$ as $|z| \to \infty$