Intermediate Quantum 491: Homework 1

1-1 Consider the set of test grades g_i = 75.71 71.43 41.43 64.29 77.14 65.71 58.57 68.57 24.29 54.29 48.57 68.57 78.57 61.43 58.57

a) Compute the mean and standard deviation.

b) Make a histogram (frequency distribution graph) with 10 bins from 0 to 100. Compute the histogram mean and uncertainty $= \sqrt{\text{variance}}$

✓ **Problem 1.3** The needle on a broken car speedometer is free to swing, and bounces perfectly off the pins at either end, so that if you give it a flick it is equally likely to come to rest at any angle between 0 and π.

(a) What is the probability density, $\rho(\theta)$? [$\rho(\theta)\,d\theta$ is the probability that the needle will come to rest between θ and $(\theta + d\theta)$.] Graph $\rho(\theta)$ as a function of θ, from $-\pi/2$ to $3\pi/2$. (Of course, part of this interval is excluded, so ρ is zero there.) Make sure that the total probability is 1.

(b) Compute $\langle\theta\rangle$, $\langle\theta^2\rangle$, and σ for this distribution.

(c) Compute $\langle\sin\theta\rangle$, $\langle\cos\theta\rangle$, and $\langle\cos^2\theta\rangle$.

✓ **Problem 1.4** We consider the same device as the previous problem, but this time we are interested in the x-coordinate of the needle point—that is, the "shadow", or "projection", of the needle on the horizontal line.

(a) What is the probability density $\rho(x)$? [$\rho(x)\,dx$ is the probability that the projection lies between x and $(x + dx)$.] Graph $\rho(x)$ as a function of x, from $-2r$ to $+2r$, where r is the length of the needle. Make sure the total probability is 1. [*Hint*: You know (from Problem 1.3) the probability that θ is in a given range; the question is, what interval dx corresponds to the interval $d\theta$?]

(b) Compute $\langle x\rangle$, $\langle x^2\rangle$, and σ for this distribution. Explain how you could have obtained these results from part (c) of Problem 1.3.

***Problem 1.6** Consider the **Gaussian** distribution

$$\rho(x) = Ae^{-\lambda(x-a)^2},$$

where A, a, and λ are constants. (Look up any integrals you need.)

(a) Use Equation 1.16 to determine A. PDF normalization to 1

(b) Find $\langle x \rangle$, $\langle x^2 \rangle$, and σ.

(c) Sketch the graph of $\rho(x)$.

***Problem 1.8** Consider the wave function

$$\Psi(x,t) = Ae^{-\lambda|x|}e^{-i\omega t},$$

where A, λ, and ω are positive real constants. [We'll see in Chapter 2 what potential (V) actually produces such a wave function.]

(a) Normalize Ψ.

(b) Determine the expectation values of x and x^2.

(c) Find the standard deviation of x. Sketch the graph of $|\Psi|^2$, as a function of x, and mark the points $(\langle x \rangle + \sigma)$ and $(\langle x \rangle - \sigma)$ to illustrate the sense in which σ represents the "spread" in x. What is the probability that the particle would be found outside this range?

b. calculate J(x,t) with wave function from previous problem

√ **Problem 1.9** Let $P_{ab}(t)$ be the probability of finding the particle in the range $(a < x < b)$, at time t.

(a) Show that

$$\frac{dP_{ab}}{dt} = J(a,t) - J(b,t)$$

where

$$J(x,t) \equiv \frac{i\hbar}{2m}\left(\Psi\frac{\partial\Psi^*}{\partial x} - \Psi^*\frac{\partial\Psi}{\partial x}\right).$$

What are the units of $J(x,t)$? [J is called the **probability current**, because it tells you the rate at which probability is "flowing" past the point x. If $P_{ab}(t)$ is increasing, then more probability is flowing into the region at one end than flows out at the other.]

/ ****Problem 1.10** Suppose you wanted to describe an **unstable particle** that spontaneously disintegrates with a "lifetime" τ. In that case the total probability of finding the particle somewhere should *not* be constant, but should decrease at (say) an exponential rate:

$$P(t) \equiv \int_{-\infty}^{+\infty} |\Psi(x,t)|^2\, dx = e^{-t/\tau}.$$

A crude way of achieving this result is as follows. In Equation 1.24 we tacitly assumed that V (the potential energy) is *real*. That is certainly reasonable, but it leads to the **conservation of probability** enshrined in Equation 1.27. What if we assign to V an imaginary part:

$$V = V_0 - i\Gamma,$$

where V_0 is the true potential energy and Γ is a positive real constant?

conservation of probability contained in probability current derived in class.

(a) Show that (in place of Equation 1.27) we now get

$$\frac{dP}{dt} = -\frac{2\Gamma}{\hbar}P.$$

(b) Solve for $P(t)$, and find the lifetime of the particle in terms of Γ.

√ ***Problem 1.12** Calculate $d\langle p\rangle/dt$. *Answer:*

$$\frac{d\langle p\rangle}{dt} = \langle -\frac{\partial V}{\partial x}\rangle. \qquad [1.38]$$

(This is known as **Ehrenfest's theorem**; it tells us that *expectation values* obey Newton's second law.)