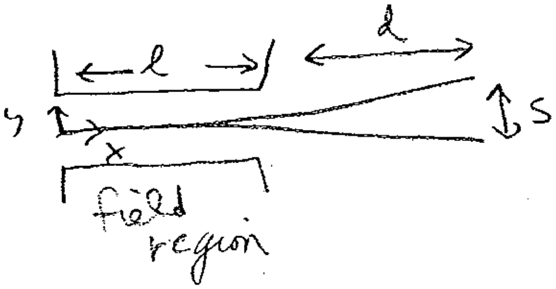
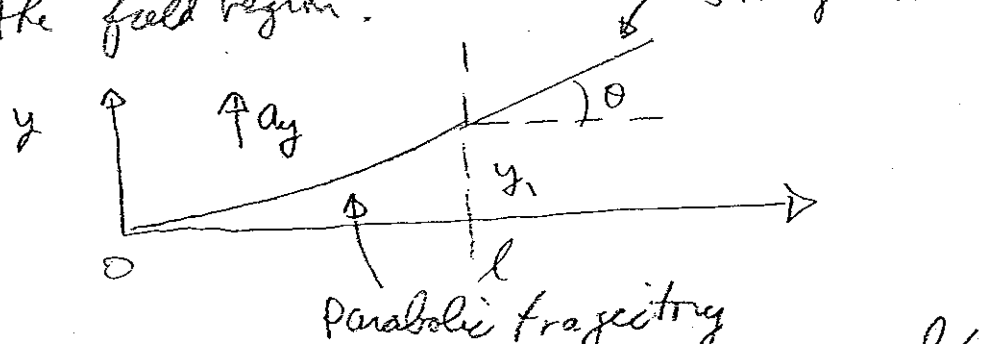
1. The atoms are uniformly accelerated inside the atom, and travel in straight lines thereafter. The acceleration, and hence required gradient, can be determined from the geometrical information.



define a coordinate system w/ $x=0$ at the start of the field region and $x=\ell$ at the end of the field region.



the particle reaches $x=\ell$ at time $t_1 = \ell/v_x$

$$y_1 = \frac{1}{2} a_y t_1^2 = \frac{1}{2} a_y \left(\frac{d}{v_x}\right)^2$$

then the geometrical condition is:

$$\tan\theta = \left.\frac{v_y}{v_x}\right|_{t_1} = \frac{s/2 - y_1}{d}$$

$$V_y(t_1) = a_y t_1 = a_y\left(\frac{\ell}{V_x}\right)$$

then solving for a_y,

$$a_y = \frac{V_x^2}{2}\frac{S}{\ell}\left(\frac{1}{d+\ell/2}\right) = V_x^2 \frac{S}{3\ell^2}$$

since $d = \ell = \frac{1}{2}$ m

now $\frac{1}{2} m V_x^2 = 2kT$, and $a_y = \frac{F_y}{m} = \frac{\mu}{m}\frac{\partial B_y}{\partial y}$

so that

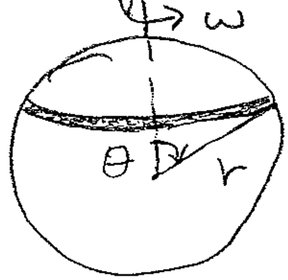
$$\frac{\partial B_y}{\partial y} = \frac{4}{3}\frac{kT}{\mu}\left(\frac{S}{\ell}\right)\frac{1}{\ell}$$

with $T = 1500$ K, $S = 10^{-3}$ m, $\ell = \frac{1}{2}$ m we have numerically, (and $k = 8.617 \times 10^{-5}$ eV/K)

$$\frac{\partial B_y}{\partial y} = \frac{4}{3}\frac{(8.62\times10^{-5}\text{ eV/K})\, 1500\text{K} \cdot 4\times10^{-3}\text{ m}^{-1}}{5.79\times10^{-5}\text{ eV/T}}$$

$$\boxed{\frac{\partial B_y}{\partial y} = \frac{8(8.62)}{5.79}\text{ T/m} = 11.9\text{ T/m}}$$

1.2 magnetic moment of solid spinning sphere (mass m) w/ surface charge q.



$$d\mu = A \frac{di}{c}$$

$$A = \pi (r \sin\theta)^2$$

$$\frac{di}{c} = \frac{dq}{c\,(\text{period})} = \frac{q}{c}\left(\frac{r^2 \sin\theta\, d\theta\, 2\pi}{4\pi r^2}\right)\frac{\omega}{2\pi}$$

$$\frac{di}{c} = \frac{q\omega}{4\pi c}\sin\theta\, d\theta \quad \text{so that}$$

$$d\mu = \pi r^2 \left(\frac{q\omega}{4\pi c}\right)\sin^3\theta\, d\theta$$

integrating w/ $x = \cos\theta$,

$$\mu = \frac{q\omega}{4c}\int_{-1}^{+1}(1-x^2)\,dx = \frac{q}{c}\frac{\omega r^2}{3}$$

then $\vec{L} = I\vec{\omega} = \frac{2}{5} m r^2 \vec{\omega}$

$$\boxed{\vec{\mu} = \left(\frac{5}{3}\right)\frac{q}{2mc}\vec{L}}$$

$$g = 5/3$$

1.4 $|\langle +x|+n\rangle|^2 = P_+^x$, the probability to measure $\hat{S}_x = +\hbar/2$.

$$|+x\rangle = \frac{1}{\sqrt{2}}\left(|+z\rangle + |-z\rangle\right)$$

$$\langle +x|+n\rangle = \frac{1}{\sqrt{2}}\left(\cos\frac{\theta}{2} + e^{i\phi}\sin\frac{\theta}{2}\right)$$

$$P_+^x = \frac{1}{2}\left(c + e^{-i\phi}s\right)\left(c + e^{i\phi}s\right)$$

$$= \frac{1}{2}\left[c^2 + s^2 + cs\left(e^{i\phi} + e^{-i\phi}\right)\right]$$

$$= \frac{1}{2}\left[1 + 2\underbrace{cs}_{2\cos\frac{\theta}{2}\sin\frac{\theta}{2} = \sin\theta}\cos\phi\right] = \frac{1}{2}\left[1 + \sin\theta\cos\phi\right]$$

$$P_-^x = 1 - P_+^x = \frac{1}{2}\left[1 - \sin\theta\cos\phi\right]$$

$$\langle \hat{S}_x\rangle = P_+^x\left(\frac{\hbar}{2}\right) + P_-^x\left(-\frac{\hbar}{2}\right) = \frac{\hbar}{2}\sin\theta\cos\phi$$

$$\langle \hat{S}_x^2\rangle = P_+^x\frac{\hbar^2}{4} + P_-^x\left(\frac{\hbar^2}{4}\right) = \frac{\hbar^2}{4}$$

$$\Delta S_x = \frac{\hbar}{2}\sqrt{1 - \sin^2\theta\cos^2\phi}$$

1.5 (a) to calculate $\langle +y|+n\rangle$

$$|+y\rangle = \frac{1}{\sqrt{2}}\left(|+z\rangle + i\,|-z\rangle\right)$$

$$\langle +y|+n\rangle = \frac{1}{\sqrt{2}}\left[\langle +z| - i\langle -z|\right]\left[\cos\frac{\theta}{2}|+z\rangle + e^{i\phi}\sin\frac{\theta}{2}|-z\rangle\right]$$

$$= \frac{1}{\sqrt{2}}\left(\cos\frac{\theta}{2} - i\,e^{i\phi}\sin\theta/2\right) = \frac{1}{\sqrt{2}}\left(\cos\frac{\theta}{2} + e^{i(\phi-\pi/2)}\sin\frac{\theta}{2}\right)$$

referring back to prob. 1.4 we see that

$$P_+^y = |\langle +y|+n\rangle|^2 = \frac{1}{2}\left(1+\sin\theta\cos(\phi-\pi/2)\right)$$

check: $\theta=0$; $P_+^y = \frac{1}{2}$ for all ϕ

$\theta=\frac{\pi}{2}, \phi=0$; $P_+^y = |\langle +y|+x\rangle|^2 = \frac{1}{2}$

$\theta=\frac{\pi}{2}, \phi=-\pi/2$; $P_+^y = |\langle +y|-y\rangle|^2 = 0$

(b) $\langle +n|+y\rangle = \langle +y|+n\rangle^* = \frac{1}{\sqrt{2}}\left(\cos\frac{\theta}{2} + e^{-i(\phi-\pi/2)}\sin\frac{\theta}{2}\right)$

$$P_+^N = |\langle +N|+y\rangle|^2 = |\langle +y|+n\rangle|^2 = \frac{1}{2}\left[1+\sin\theta\cos(\phi-\pi/2)\right]$$

1.6 Recall how the dual bra is defined with complex conjugation of components –

$$|+n\rangle = \cos\frac{\theta}{2}\,|+z\rangle + e^{i\phi}\sin\frac{\theta}{2}\,|-z\rangle$$

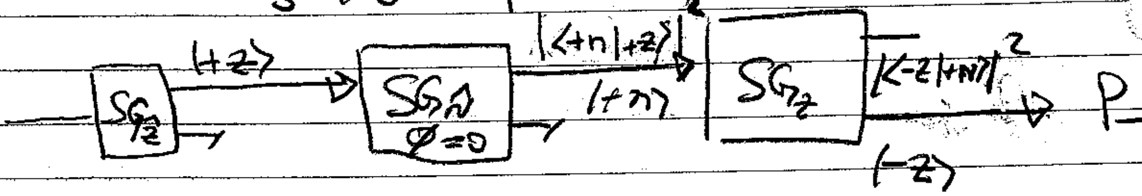
$$|-n\rangle = \sin\frac{\theta}{2}\,|+z\rangle - e^{i\phi}\cos\frac{\theta}{2}\,|+z\rangle$$

$$\langle +n|-n\rangle = \left(c\langle +z| + e^{-i\phi} s\langle -z|\right) \cdot \left(s\,|+z\rangle - e^{i\phi} c\,|-z\rangle\right)$$

$$= cs - cs = 0$$

$$\langle -n|-n\rangle = \left(s\langle +z| - e^{-i\phi} c\langle -z|\right)\left(s\,|+z\rangle - e^{i\phi} c\,|-z\rangle\right)$$

$$= s^2 + c^2 = 1$$

1.7 

(a) $|+n(\theta,0)\rangle = \cos\frac{\theta}{2}\,|+z\rangle + \sin\frac{\theta}{2}\,|-z\rangle$

$$\langle +n|+z\rangle = \langle +z|+n\rangle^* = \cos\frac{\theta}{2}$$

$$\langle -z|+n\rangle = \sin\frac{\theta}{2}$$

The probability P_- is the *product* of these amplitudes squared:

$$P_- = \cos^2\frac{\theta}{2}\sin^2\frac{\theta}{2} = \frac{1}{4}\sin^2\theta$$

(b) max P_- is for $\theta = \pm\frac{\pi}{2}$, $P_-^{max} = \frac{1}{4}$

(c) Without SG_N intermediate measurement, $P_- = 0$.

1.8 $|\psi\rangle = \frac{i}{\sqrt{3}}|+z\rangle + \sqrt{\frac{2}{3}}|-z\rangle$

P_+ = probability to measure $+\frac{\hbar}{2} = \left|\frac{i}{\sqrt{3}}\right|^2 = \frac{1}{3}$

$P_- = \left|\sqrt{\frac{2}{3}}\right|^2 = \frac{2}{3}$

$\langle S_z\rangle = \frac{\hbar}{2}\left(\frac{1}{3} - \frac{2}{3}\right) = -\frac{1}{3}\frac{\hbar}{2}$

$\langle S_z^2\rangle = \frac{\hbar^2}{4}\left(\frac{1}{3} + \frac{2}{3}\right) = \frac{\hbar^2}{4}$

$\Delta S_z = \sqrt{\langle S_z^2\rangle - \langle S_z\rangle^2} = \frac{\hbar}{2}\sqrt{1 - \frac{1}{9}} = \frac{\sqrt{2}}{3}\hbar$

The experiment would measure

$N_+ \equiv$ # spin up ; $N_- =$ # spin down

$N_+ + N_- = N$ = # measurements

the fluctuations are binomial.

$\sigma^2 = N P_+ P_- = N\frac{2}{9}$, $\sigma = \sqrt{N}\sqrt{\frac{\sqrt{2}}{3}} \cong \sqrt{N}\, 0.7$

reasonable data would look like $\boxed{N_+ = NP_+ \pm \sigma = \frac{N}{3} \pm 0.7\sqrt{N}}$

N	10^2	10^3	10^4	
NP_+	33	333	3333	mean
σ	7	22	70	1σ fluctuation
σ/NP_+	21%	7%	2%	fractional error